ARE GLUEBALLS FOUND?

by

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Submitted to the Princeton University Physics Department in partial fulfillment of the requirements for the degree of Bachelor of Arts

April 1981

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I. Introduction

In the beginning God created Yang and Mills and in 1954 Yang and Mills[1] created non-Abelian gauge theories. Yet in the beginning only the best men believed and put faith in the new savior. Today these select few reap the benefits[2] of their early commitment. The pagan world remained oblivious to the Word until a generation had passed and 't Hooft[3] established the renormalizibility of the theories. Soon after this, Gross and Wilczek[4] discovered asymptotic freedom, which not only helped to explain the phenomenon of Bjorken scaling in deep inelastic scattering experiments, but also firmly established quantum chromodynamics, the non-Abelian gauge theory based upon the unbroken SU(3) color symmetry, as the proper description of the strong interactions. Because of its many successes in the past decade, QCD has acquired a large following of theoretical and experimental physicists, all of whom believe. Recent Monte Carlo work by Creutz[5] has revealed the compatibility of asymptotic freedom and confinement for non-Abelian gauge theories, thereby insuring that QCD provides a correct description in both the ultraviolet and infrared sectors. Furthermore, investigations by Callan, Dashen and Gross[6] give evidence for the fundamental role of instantons in the intermediate coupling regime where a gap in the quantum chromodynamic β-function is observed in the rapid transition from strong to weak coupling. Soon our understanding of the strong interactions should be complete.

If the current wisdom holds true, there should exist color singlet bound states of gluons, called glueballs, which are a consequence of the gauge particle self-couplings permitted in the non-Abelian theory. At the present moment not a single glueball resonance has been firmly established via accelerator experiments- a fact that causes high energy theorists some minor discomfort. Yet the inescapable truth is that QCD cannot survive unless the glueballs are found. Here we examine the most recent theoretical and numerical analyses of the glueball mass spectrum and review the basic experimental situation regarding possible glueball candidates. In addition, a strong coupling expansion for the mass gap of the three dimensional Euclidean lattice gauge theory is performed using the techniques of Munster[7]. Overall, the situation is discovered to be rather severe as the theorists remain unable to explain the absence of gluonic bound state resonances. Finally, suggestions are made for future experimental searches and theoretical investigations of the glueball mass spectrum.

II. Classical Glueballs

Amidst the fury of interest in classical Yang-Mills solutions that occurred in the middle 70's, Coleman[8] was able to prove that classical glueballs did not exist and if there were glueballs in the real world they were a manifestation of the quantum nature of QCD. He originally conceived of the classical glueball as a "lump" of energy held together by its own self interaction- a result of the nonlinear nature of the theory. Earlier work by Coleman[9], Deser[10], and Pagels[11] had revealed that, for the classical Yang-Mills theory in four dimensional Minkowski space-time, there were no finite-energy non-singular time independent or time periodic lumps. A most elegant proof by Coleman made it apparent that the above arguments could be generalized so that regardless of their time dependence, classical lumps did not exist, but always quickly radiated away their energy to spatial infinity. After Coleman we note that consideration of the standard Yang-Mills field strength tensor,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + c^{abc} A^b_\mu A^c_\nu \qquad \begin{array}{l} \mu = 0,1,2,3 \\ a = 1,\ldots,N^2-1 \text{ for } SU(N) \end{array}$$

where A^a_μ are the vector gauge fields and c^{abc} are the structure constants of the Lie group, as well as the related energy-momentum tensor

$$\Theta^{\mu\nu} = F^{\mu\gamma a} F_\gamma^{\ \nu a} - \frac{1}{4} g^{\mu\nu} F^a_{\gamma\sigma} F^{\sigma\gamma a}$$

and the conventionally defined Yang-Mills electric and magnetic fields,

$$E^a_i = F^a_{i0} \quad , \quad H^a_i = \frac{1}{2}\epsilon_{ijk} F^a_{jk}$$

leads one to the result that

$$|e_i \Theta^{0i}| \leqslant \Theta^{00} \qquad (*)$$

where $\hat{e}$ is an arbitrary vector in space, since $\Theta^{00} = \frac{1}{2}(\vec{E}^a \cdot \vec{E}^a + \vec{H}^a \cdot \vec{H}^a)$ and $\Theta^{0i} = (\vec{E}^a \times \vec{H}^a)^i$. As the space integral of Θ^{00} and $|\Theta^{0i}|$ represent, respectively, the total energy and momentum of the lump, it is clear that the classical glueball must travel at a velocity, $|\vec{v}| = \frac{|\vec{P}|}{E}$, less than or equal to the speed of light.

The fact that classical glueballs traveling at sub-light speeds do not exist is easily shown by Lorentz transforming to the rest frame of the hypothetical finite-energy non-singular lump. Defining

$$F(r,t) = \int_{|\vec{x}| \leqslant r} d^3x \, x_i \Theta^{0i}$$

the zero divergence and traceless nature of the energy-momentum tensor imply

$$\partial_0 F(r,t) = \int_{|\vec{x}| \leqslant r} d^3x \, \Theta^{00} + \int_{|\vec{x}| = r} d^2S_i x_j \Theta^{ij} \qquad (\hat{x} = d^2S_i)$$

Our assumption that the classical lump has finite energy E and is nonradiating dictates that, as $r \to \infty$, the RHS of the above equation goes to E, uniformly in time (t>0). This in turn implies that there exists an r such that

$$\partial_0 F(r,t) \geqslant \frac{E}{2}$$

so that at this distance, for all future times,

$$F(r,t) \geqslant \frac{E}{2} t + F(r,0)$$

But our previous result (*), requires

$$|F(r,t)| \leqslant r \int_{|\vec{x}| \leqslant r} d^3x \, \Theta^{00} \leqslant rE$$

so that the two inequalities are satisfied only if E=0; that is, if the lump is nothing more than the trivial vacuum solution.

Finally, the proof that lightlike classical glueballs are not to be found simply entails gauging away all relevant degrees of freedom till one is left with just the vacuum. Choosing the velocity vector of the classical lump to be pointed in the x^3-direction, one obtains transversality equations analogous to those in electromagnetism for a free traveling plane wave,

$$\vec{H}^a = \hat{k} \times \vec{E}^a \quad , \quad \vec{E}^a = -\hat{k} \times \vec{H}^a \qquad (\uparrow \hat{x}^3, \hat{k})$$

Introduction of the light cone variables, $x^{\pm} = x^0 \pm x^3$, allows one to rewrite the above as

$$F^a_{-\mu} = 0 \qquad (**)$$

and

$$F^a_{12} = 0$$

This last equation implies that one can choose a gauge such that $A^a_1 = A^a_2 = 0$. The A^a_- is gauged away by noting that, as the 1,2 components are now identically set to zero, (**) reveals A^a_- to be independent of x^1 and x^2. Hence, a gauge can be picked so that $A^a_- = 0$ as well. Lastly, to gauge away A^a_+ we utilize the Yang-Mills equation of motion

$$\partial^\mu F^a_{\mu\nu} + c^{abc} A^{\mu b} F^c_{\mu\nu} = 0$$

put $\nu = +$, and observe the vast number of zero terms dropping out so that we are left with

$$(\partial^1 \partial_1 + \partial^2 \partial_2) A^a_+ = 0$$

As we stipulated that our classical glueball be a finite-energy non-singular solution, the above equation requires A^a_+ to be independent of x^1 and x^2. Furthermore, from (**) we know that A^a_+ is also independent of x^-. Thus, since it is clear that A^a_+ can only depend on x^+, we can now employ our last permissible gauge transformation to gauge away A^a_+ too.

As was very aptly pointed out by Coleman, the nonexistence of Yang-Mills lumps says very little about the possibility of quantum glueballs and, if QCD is the correct theory of the strong interactions, the glueballs of the real world will be color singlet, flavorless, electrically neutral bound states of gluons, the gauge field quanta of the model.

III. The Glueball Mass Spectrum

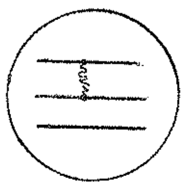
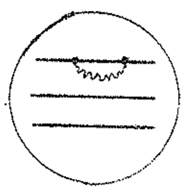
a. MIT Bag Model

As perhaps the most successful phenomenological model of hadronic structure proposed to this date, the MIT Bag Model, conceived by Chodos, Jaffe, Johnson, Thorn and Weisskopf[12] in the year 1974, was a crucial step beyond the general constituent quark models of the 60's since its creators considered a theoretical construct in which the component quarks and gluons were relativistic in nature. In fact, the fundamental assumption made by the MIT group was that a hadron was nothing more than a confined region of space where the quark and gluonic fields existed. The confinement mechanism was manifested by supplying the region, or "bag" as it were, with a finite energy B per unit volume. In their subsequent calculations, for which they considered massless spin $\frac{1}{2}$ colored quarks interacting with the eight massless vector bosons of the SU(3) color gauge theory, Chodos, Jaffe, Johnson and Thorn[13] discovered that the imposition of boundary conditions on their phenomenological bag incurred the requirement that all hadrons exist in color singlet zero triality states. Moreover, solving the problem via a classical approximation scheme, since an exact solution was not possible, the group found that it was plausible to assume the bag to be of spherical structure. A further restriction imposed by the boundary conditions of the bag stipulated that the constituent quark fields occupy only those modes with angular momentum $j=\frac{1}{2}$. Initial application of their bag model hypotheses in the baryonic realm proved most promising, as Chodos, Jaffe, Johnson and Thorn were able to compute very acceptable values for the gyromagnetic ratio, axial-vector acceptable values for the gyromagnetic ratio, axial-vector charge and charge radius of the proton, in addition to successfully constructing a reasonable, though somewhat degenerate, classification scheme for the low-lying nonstrange baryon states.

Taking note of the moderate, but quite surprising, first triumphs of the MIT bag model, DeGrand, Jaffe, Johnson, and Kiskis[14] continued investigations along the lines of the original group and were able to greatly refine the earlier technique. A crucial aspect of this new work involved the construction of a complete hadronic mass Hamiltonian that described the entire system consisting of quark fields, gluons and the bag. The MIT group proposed that the appropriate Hamiltonian was of the form,

$$M(R) = E_v + E_0 + E_q + E_m + E_e$$

where $E_v = \frac{4}{3}\pi R^3 B$ and $E_0 = -\frac{Z_0}{R}$ were, respectively, the total energy due to the necessary confining bag pressure and the zero point energy of all the cavity modes. E_q , the quark kinetic energy term responsible for approximately 75% of the mass of the hadron, was a relatively complicated expression dependent upon an eigenvalue equation derived from the boundary conditions. Inclusion of the lowest order gluon interaction graphs,

exchange   self energy

resulted in the contribution of the last two terms in the hadronic mass Hamiltonian. $E_m = \sum_{i>j} (\vec{\sigma}_i \cdot \vec{\sigma}_j) M_{ij}$, where M_{ij} was a magnetic interaction strength term, linear in the quark gluoncoupling constant, that was determined by an integral over the bag wave functions, represented the very important color magnetic exchange term. Lastly, the color electric energy, in its entirety, was taken into account by the addition of the last term E_e. To actually calculate hadron masses, the MIT group first constructed overall antisymmetric states, recalling color, flavor, and spin degrees of freedom, since the quarks themselves obeyed Fermi-Dirac statistics, then diagonalized the energy Hamiltonian in this basis, and finally minimized the energy eigenvalues with respect to the cavity radius R for each eigenstate. This methodology was a consequence of the fact that, for the required $j=\frac{1}{2}$ quarks, a second, nonlinear, boundary condition on the bag required that the quark and gluonic field pressures locally balance the bag pressure B at the surface. Under the assumption of zero nonstrange quark mass and employing the known masses of the N, Δ, ω, and Ω^- to fix the adjustable parameters B, α_c, Z_0 and m_s, the theorists found remarkable agreement between observed and calculated hadron masses (cf. table 1).

table 1.

TABLE 1. Masses of light hadrons for the case $m_0=0$. All masses are quoted in GeV, R_0 in GeV^{-1}. The five contributions to the hadron mass, E_V, E_0, E_Q, E_M, and E_E, defined by Eq. (3.6) and preceding equations, are listed and compared with experiment.

Particle	M_{exp}	M_{bag}	R_0	E_0	E_V	E_Q	E_M	E_E
p	0.938	0.938	5.00	−0.367	0.234	1.226	−0.155	0
Λ	1.116	1.105	4.95	−0.371	0.227	1.400	−0.156	0.005
Σ^+	1.189	1.144	4.95	−0.371	0.227	1.400	−0.116	0.005
Ξ^0	1.321	1.289	4.91	−0.374	0.222	1.572	−0.136	0.005
Δ	1.236	1.233	5.48	−0.336	0.308	1.119	0.141	0
Σ^*	1.385	1.382	5.43	−0.338	0.301	1.292	0.122	0.005
Ξ^*	1.533	1.529	5.39	−0.341	0.293	1.465	0.106	0.005
Ω^-	1.672	1.672	5.35	−0.343	0.287	1.636	0.092	0
ρ	0.77 ± 0.01	0.783	4.71	−0.390	0.196	0.868	0.110	0
K^*	0.892	0.928	4.65	−0.395	0.189	1.039	0.091	0.004
ω	0.783	0.783	4.71	−0.390	0.196	0.868	0.110	0
ϕ	1.019	1.068	4.61	−0.399	0.183	1.207	0.076	0
K	0.495	0.497	3.26	−0.564	0.065	1.407	−0.415	0.003
π	0.139	0.280	3.34	−0.549	0.070	1.222	−0.462	0

$B^{1/4}=0.145$ GeV, $Z_0=1.84$, $\alpha_c=0.55$, $m_s=0.279$ GeV

A schematic plot makes clear the good numbers produced by their hadronic mass Hamiltonian (cf. fig.1):

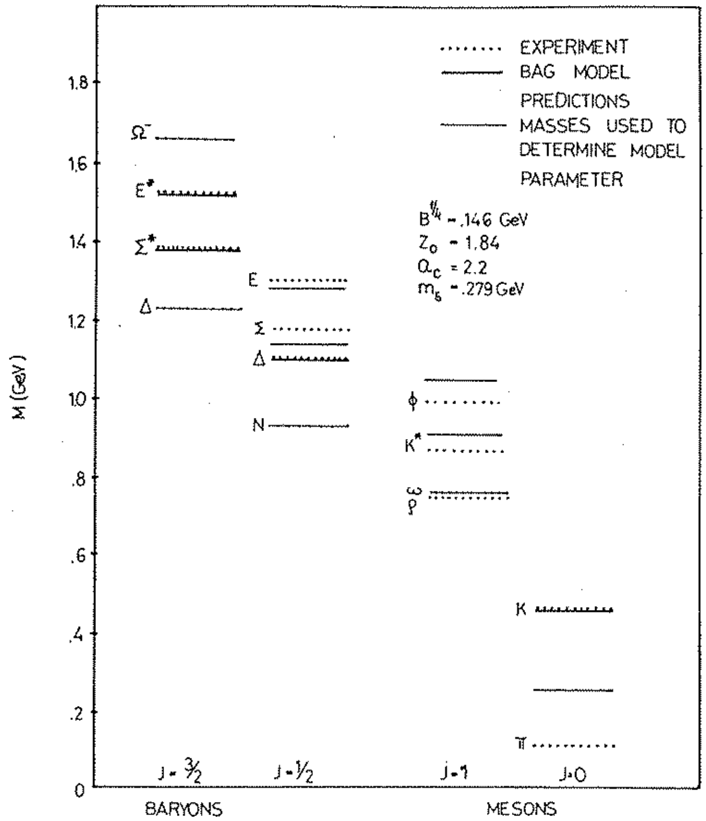


fig. 1.

Fig. 7.2. The M.I.T. fit to the hadron masses where the non-strange quarks are assumed to be massless. The experimental values are given by dotted lines for comparison. There are four parameters B, α_c, Z_0 and m_s (strange quark mass) to fit. The masses of the N, Δ, ω, and Ω were used to determine the parameters.

By adjusting their theoretical computations to accomodate u and d quark masses the order of 100 MeV, the MIT group furthermore discovered that such a minor correction had negligible effect upon their mass results. Marked by these successes, the relativistic bag model of hadronic structure soon superseded all previous nonrelativistic explanations.

Towards the close of 1975, Jaffe and Johnson[15], two of the MIT theorists responsible for the initial introduction and further investigation of the phenomenological bag model, became interested in the possible existence of unconventional bound states of quarks and gluons. Among the unusual systems postulated by the MIT pair were gluonic hadrons- quarkless elementary particles consisting of nothing more than several gluons in a confined color singlet state. Employing the standard MIT bag model methodology, Jaffe and Johnson constructed quantum glueballs by populating the modes of the gluon field in a spherical cavity with wave functions which satisfied the bag boundary conditions necessary to insure confinement. Via the mandate of asymptotic freedom, they concluded that for short distances the order of 1 fermi (the bag size), $\alpha_c \propto g^2$ was small so that in a first approximation, the gluon self-coupling could be ignored. Removal of the gauge boson self-couplings relegated the confined color vector gluons to the status of mere Abelian vector fields which satisfied Maxwell's equations inside the static bag and were subject to the boundary conditions:

$$\vec{n} \cdot \vec{E}^a = 0$$

and

$$\vec{n} \times \vec{B}^a = 0$$

Solution of the single gluon mode wave function thence became a simple matter, nothing more than a standard problem in electrodynamics- the confinement of a massless vector field to a spherical cavity. Jaffe and Johnson discovered that for each value of total angular momentum $J \geqslant 1$, there existed two types of gluonic modes- The transverse electric (TE) mode with parity $\pi = (-1)^{J+1}$ and the transverse magnetic (TM) mode with parity $\pi = (-1)^J$. For each value of J, π there was an infinite sequence of higher energy modes with increasing numbers of radial nodes. There were no zero orbital angular momentum modes and the lowest states were the $l=1$ TE mode (1^+), with energy $E = 2.744/R$, and $l=1$ TM mode (1^-) with associated energy $E = 4.49/R$. The MIT theorists constructed two gluon color singlet bound states with charge conjugation C=+1 via the coupling δ_{ab} and glueballs composed of three gluons by means of the couplings d_{abc}, f_{abc} which dictated charge conjugation quantum numbers +1,-1 respectively. The calculations of Jaffe and Johnson in the glueball sector revealed the following distribution of lowest lying states,

MODE	$J^{\pi C}$	MASS (MeV)
$(TE)^2$	$0^{++}, 2^{++}$	960
$(TETM)$	$0^{-+}, 1^{-+}, 2^{-+}$	1290
$(TE)^3$	$0^{+-}, 1^{++}, 2^{+-}, 3^{++}$	1460
$(TM)^2$	$0^{++}, 2^{++}$	1590

all of which were color and flavor singlets and involved only the lowest angular momentum TE and TM modes.

It is crucial at this point to enumerate the various limitations inherent to the MIT bag model computation of glueball masses. Firstly, as the lowest order gluon exchange corrections were not yet incorporated into the glueball model by Jaffe and Johnson, their mass spectrum was plagued by degeneracies expected of an approximation in which color magnetic spin-spin splittings are neglected. Secondly, at the semiclassical level, the gluonic bound states of nonzero J are not, in fact, spherical, so that one cannot trust with great confidence the MIT bag model estimates for their masses. Lastly, the theorists assume, as can only be expected of them, in this, the first real effort to explore the glueball mass spectrum, that the gluonic hadrons are pure and that there is no mixing with ordinary hadrons of identical quantum numbers. Despite these apparent shortcomings in their phenomenological bag model calculation of glueball masses, Jaffe and Johnson nonetheless conclude that there should be a veritable plethora of gluonic bound states in the region 1-2 GeV.

b. Glue Lego.

In 1977, Robson[16], inspired perhaps by both the simplicity and success of the constituent quark models of the 1960's, advocated a somewhat more direct approach to the problem of the glueball spectrum. He envisioned the gluons, quite literally, as the 'building blocks' out of which the gluonic bound states were formed. Tagging the component gluons with two labels, one to locate its position in the color octet and the other to identify its spin polarization state, Robson merely constructed color singlet states that were symmetric on the interchange of any two gluon indices, since the gauge quanta, themselves, obeyed Bose-Einstein statistics. Thus, for a glueball composed of two gluons, one merely notes that the identity irreducible representation is contained in the Clebsch-Gordan decomposition series resulting from the direct product of the adjoint rep with itself:

$$\{8\} \otimes \{8\} = \{1\} \oplus \{8\} \oplus \{8\} \oplus \{10\} \oplus \{\overline{10}\} \oplus \{27\}$$

and that the Bose symmetry condition for the color singlet requires

$$+ = \pi_1 \pi_2 (-1)^L (-1)^{S-S_1-S_2} I$$

Because $\pi_1 = \pi_2 = -1$, $S_1 = S_2 = 1$ for the vector gluons, whilst I=1 for the interchange of color labels in the singlet state, this in turn implies

$$+ = (-1)^{L+S}$$

so that (L+S) is even for 2g bound states. Such reasoning results in the following states:

Table 2
Glue lego states of given orbital angular momentum

L	J^{PC}
0	$0^{++}; 2^{++}$
1	$0^{-+}; 1^{-+}; 2^{-+}$
2	$2^{++}; 0^{++}, 1^{++}, 2^{++}, 3^{++}, 4^{++}$
3	$2^{-+}, 3^{-+}, 4^{-+}$

all of which are consistent with the MIT bag model predictions for possible two gluon glueballs.

With regard to the construction of 3g glueballs, Robson contends that the space and charge conjugation parity assignments of Jaffe and Johnson are in error because they neglect the multiplicative contributions coming from the gluons themselves. Recalling that one produced a color singlet state in this sector via either antisymmetric f-type or symmetric d-type coupling, Robson wrote the generic wave functions,

$$\Psi_d^{ijk} = d_{abc} A_a^i A_b^j A_c^k$$
$$\Psi_f^{ijk} = f_{abc} A_a^i A_b^j A_c^k$$

where the indices i,j,k labeled the spins of the gluons. Now, however, the hermitian properties of f_{abc}, d_{abc}, together with the intrinsic negative charge parity of the gluon dictated that the glueball state built with d-type coupling possess C=-1, while those gluonic bound states constructed with f-type coupling have C=+1. Under the assumption that the ground state of a three gluon system has a spatial wave function that is symmetric on the interchange of any two position labels one must consider the two possibilities:

SPACE	SPIN	COLOR
S	A	A(f)
S	S	S(d) ← coupling

Noting first that one can get seven spin states from three spin 1 distinguishable particles ($\{1\}\otimes\{1\}\otimes\{1\} = \{0\}\oplus 3\cdot\{1\}\oplus 2\cdot\{2\}\oplus\{3\}$) we next, for pictorial purposes, associate the 27 spin index degrees of freedom of the tensor ψ^{ijk} with the lattice points of a cube of side three. Symmetrizing on the cyclic permutation $i \rightarrow j \rightarrow k$ removes 16 of the 27 degrees of freedom, leaving us with 11 (cf. fig. 2).

fig. 2.

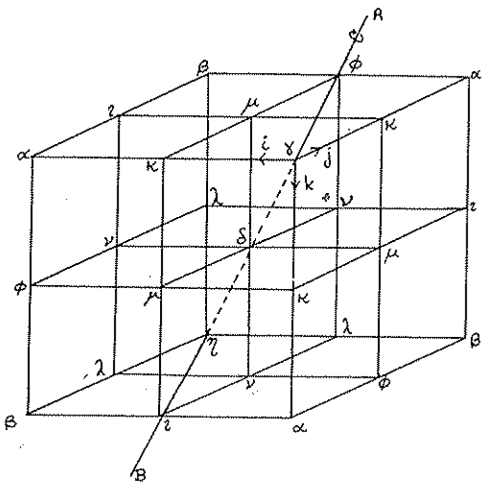


Fig. 2. The spin wave function ψ_{ijk} $(i, j, k = -1, 0, +1)$ is represented by a cube. The requirement of permutational symmetry on $i \rightarrow j \rightarrow k \rightarrow i$ is equivalent to 120° rotational symmetry about AB, and reduces the 27 degrees of freedom to 11, denoted by $\alpha, \beta, \gamma, \delta, \eta, \phi, \iota, \kappa, \lambda, \mu, \nu$.

We can now construct the antisymmetric and symmetric spin parts alluded to above. Antisymmetrization on $i \rightarrow j$ leaves just a single degree of freedom, which corresponds to the spin zero particle ($\{0\}$) with overall wave function:

$$|0^{-+}\rangle = \epsilon_{ijk} f_{abc} A^i_a A^j_b A^k_c$$

since $\Pi^{J=0}_{3\,\mathrm{gluons}}=(-1)^3=-1$. Likewise, symmetrization on $i \longrightarrow j$ yields, not unexpectedly, ten degrees of freedom (with the constraint $\varphi=1$, cf. above figure). To choose between the possibilities of two spin 2 states or one spin 1 and one spin 3 state, alternatives both of which yield the necessary ten degrees of freedom, we merely note that the point γ ($i=j=k=-1$) corresponds to a spin 3 particle. Hence, Robson concluded that there were but three zero orbital angular momentum 3g glueball states: $J^{\pi C}=0^{-+}, 1^{--}, 3^{--}$. Whereas he conjectured the last to be the mother of the Pomeron, Robson was certain that the vector glueball was none other than the first daughter of the Pomeron.

c. A Scalar Gluonium Mass Estimate Via QCD Low Energy Theorems

Shifman[17], operating entirely within a framework in which Quantum Chromodynamics is <u>the</u> quantum field theoretic description of the strong interactions, has very recently attempted to calculate the mass of the lowest lying glueball by using low energy theorems proved just last year by Soviet physicists. Motivated by the thought that quarkless QCD should be a simpler problem to tackle than the full fledged gauge theory replete with fermions, yet aware of the fact that the notion of a constituent building block gluon was somewhat preposterous, Shifman proposed that more reasonable and reliable glueball estimates could be made if one dealt with current gluons which could be well defined via the operator formalism of QCD. In particular, Shifman emphasized the importance of the 2-point function of gluonic currents, which could yield much information on gluonic mesons. If one considered the $J^{\pi C}=0^{++}$ 2-point function of gluonic currents, there existed a low energy theorem, due to Voloshin and Zakharov[18], which first permitted one a rough picture of the corresponding spectral density, and then a second low energy theorem, of Novikov et al.[19], which allowed one to actually estimate the mass of a 0^{++} gluonium state. After Shifman, we note that the object of interest is the correlator,

$$\Pi(q^2) = i\int e^{iqx}dx\,\langle 0|T\{j(x),j(0)\}|0\rangle$$

with

$$j = \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu}G^a_{\mu\nu}$$

$$\beta(\alpha_s) = -\frac{f\alpha_s^2}{2\pi} + \mathcal{O}(\alpha_s^3)$$

where $G^a_{\mu\nu}$, of course, is the field strength tensor, $\beta(\alpha_s)$ is the Gell-Mann-Low function, and $f=(\frac{11}{3}N_c - \frac{2}{3}N_f)$ is a factor dependent upon the number of colors and flavors in the theory. As a consequence of the QCD low energy theorem of Voloshin and Zakharov, one has

$$\langle 0|j|p\bar{p};\text{total momentum } q\rangle = q^2 + \mathcal{O}(q^4) \qquad p=\pi, K \text{ or } \eta \text{ meson}$$

so that in the chiral limit (m_q=0) for small q^2, where we expect critical nonperturbative effects to be important, it is apparent that the imaginary part of the correlator is 'saturated' by the Goldstone mesons 2π, 2K, 2η:

$$\mathrm{Im}\,\Pi(q^2) = \frac{N_f^2-1}{2}\,\frac{q^4}{16\pi} + \mathcal{O}(q^6) \qquad (*)$$

As pointed out by Shifman, the key fact here is that although the current, as defined before, is proportional to α_s, the Goldstone pair contribution to $\mathrm{Im}\,\Pi(q^2)$ is much greater than the perturbative result for the same quantity:

$$\mathrm{Im}\,\Pi^{PERT}(q^2) = \frac{2}{\pi} q^4 \left[\frac{f\alpha_s}{8\pi}\right]^2 + \mathcal{O}(\alpha_s^3)$$

A plot of the imaginary part of the correlator reveals an enhancement (cf. fig. 3)

fig. 3.

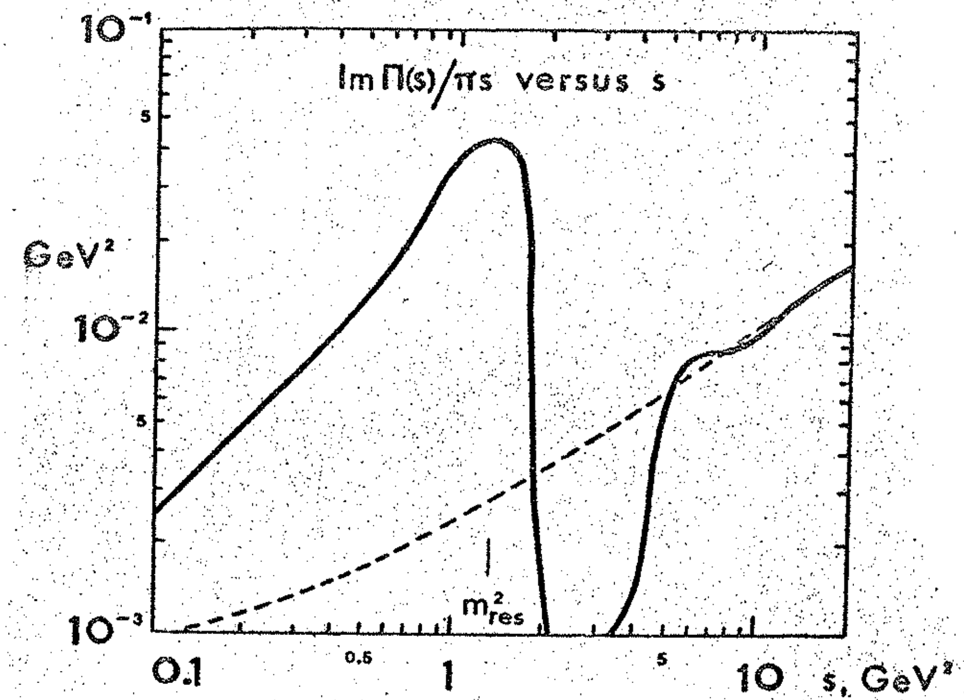


fig. 3. The imaginary part of the two-point function (1) in the chiral limit. Solid curve: the physical spectral density (schematic); dashed curve: the parton-like prediction (in calculating $\mathrm{Im}\,\Pi^{pert}$ we accept the following value for the QCD parameter Λ which fixes $\alpha_s(s)$: $\Lambda = 100$ MeV).

One expects that at larger q^2, this enhancement will develop into a resonance structure which one can associate with a physical gluonium state. Nevertheless, since there will most assuredly be substantial mixing between quark and gluonic bound states of identical quantum numbers, it is extremely unlikely that the expected peak will be a single gluonic resonance of the standard Breit-Wigner variety.

To estimate the mass of the above $J^{PC}=0^{++}$ gluonium resonance, Shifman utilized the QCD low energy theorem established by Novikov's group, which relates the zero momentum point of the correlator to the vacuum expectation value of $\alpha_s G^2$:

$$\Pi(q^2=0) = -4\langle 0|\frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$$

From the work of Vainshtein et al.[20], Shifman knew that the above relation was equivalent to the dispersion sum rule

$$\frac{1}{\pi}\int_0^\infty \frac{ds}{s}\left(\mathrm{Im}\,\Pi(s) - \mathrm{Im}\,\Pi^{PERT}(s)\right) = 16|\varepsilon_{vac}|$$

where $\varepsilon_{vac} = -\frac{1}{4}\langle 0|\alpha_s G^2|0\rangle$ is the vacuum energy density associated with the nonperturbative fluctuations of QCD. In the low energy approximation of this approach, one can neglect $\mathrm{Im}\,\Pi^{PERT}(s)$, so that substitution of (*) into the above yields,

$$\frac{N_f^2-1}{64\pi^2} S_r^2 = 16\,|\varepsilon_{vac}^{ch}|$$

where we necessarily substituted the chiral limit vacuum energy density and $s=s^r$ is the critical resonance position at which the function $\frac{\mathrm{Im}\,\Pi(s)}{\pi s}$ deviates significantly from a straight line. Since this point locates the resonance,

$$m^2_{RES} = S_r = 32\pi\sqrt{\frac{|\varepsilon_{vac}^{ch}|}{N_f^2-1}}$$

Plugging in the accepted value $\varepsilon_{vac}^{ch} \approx -\frac{1}{2}(240\ \mathrm{MeV})^4$ and assuming a world of three quark flavors, Shifman obtained the following estimate for the lowest lying scalar gluonium state:

$$m_{RES}^{ch} \approx 1.2 \text{ GeV}$$

A more refined analysis by Shifman, in which he respects the fact that an enhancement in the spectral density is inevitably accompanied by a subsequent gap (mathematically, this amounts to incorporating the duality relation $\frac{1}{\pi}\int_0^\infty ds\left(\mathrm{Im}\,\Pi(s) - \mathrm{Im}\,\Pi^{PERT}(s)\right) \approx 0$ into the calculation and insuring that it, as well as the above dispersion sum rule, is satisfied by Π^{PHYS}), alters the first value by 10%:

$$m_{RES}^{ch} \approx 1.3 \text{ GeV}$$

Finally, in the limit of the real world ($m_u=m_d=0, m_s \approx 150$ MeV), the gluonium mass estimate is pushed up another 100 MeV:

$$m_{scalar} \approx 1.4 \text{ GeV}$$

As a result of the above analysis, Shifman suggests that the $\varepsilon(1300)$ seen in S-wave $\pi\pi$ scattering is nothing more than the superposition of two glueballs.

d. Boxiton States in the Hamiltonian Formulation of Lattice QCD

Here we investigate the use of strong coupling expansions to estimate glueball mass ratios in the Hamiltonian version of the 3+1 dimensional SU(3) lattice gauge theory. After Kogut, Sinclair and Susskind[21], we consider the gauge-invariant lattice Hamiltonian,

$$H_{gauge} = \frac{g^2}{2a}\left\{\sum_{links}\left[E(r,\lambda)\right]^2 - \frac{2}{g^4}\sum_{plaquettes}\left(\mathrm{Tr}\, UUUU + h.c.\right)\right\}$$

where the first term, the "chromo-electrostatic" contribution, is the sum over the entire lattice of the link SU(3) quadratic Casimirs, while the second term is the important "chromo-magnetokinetic" contribution, which involves the sum over all plaquettes of the

Wilson trace. In the following, we will find it simpler to deal with the effective "Wamiltonian", defined by Kogut[22] as,

$$W = \frac{2a}{g^2} H_{gauge} = \sum_{links} E^2 - y \sum_{plaquettes} (\mathrm{Tr}\, UUUU + h.c.) \equiv W_0 - y W_{kinetic} \qquad (*)$$

where $y = \frac{2}{g^4}$ is the characteristic perturbative expansion parameter that we will use later. In the strong coupling limit of the lattice theory, it is clear that the vacuum $|0\rangle$ is that state with the property

$$E^2(r,\hat{n})\,|0\rangle = 0$$

for all the links of the system, so that the ground state is characterized by vanishing fluctuations of the non-Abelian electric field. In this limit the lowest energy excitations of the vacuum (i.e. the particles in the Hamiltonian formulation of lattice QCD) are oriented flux loops, called boxitons, obtained by applying the Wilson trace operator to $|0\rangle$. For example,

$$\{\mathrm{Tr}\, U(1)U(2)U(3)U(4)\}|0\rangle$$

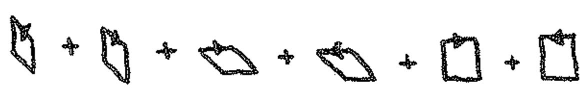
where the links 1,2,3,4 are those associated with a single plaquette $\left(2\,\underset{1}{\overset{3}{\square}}\,4\right)$, represents the first excited state of the system. It has an energy $\omega = 4 \times \frac{4}{3}$ in the strong coupling limit since for y=0, the Wamiltonian receives a contribution of $\frac{4}{3}$ from the chromo-electrostatic quadratic Casimir for each $(3,\bar{3})$ link in this boxiton.

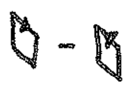
We note that because of our approach thus far, the boxitons are position eigenstates labeled by their location in the lattice and their orientation. Position states are just fine provided that we're in the deep strong coupling regime where the boxiton merely sits in the lattice, receiving no kinetic energy contribution to its total energy since the chromo-magnetokinetic term is wiped out in this limit. However, for nonzero y, the chromo-magnetokinetic term of the Wamiltonian induces translations of the boxiton in the lattice so that it will be advantageous for us to consider momentum eigenstate boxitons,

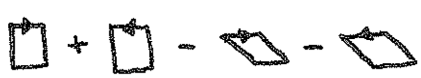
$$|B(k)\rangle = \sum_{r=(x,y,z)} e^{i\vec{k}\cdot\vec{r}}\, |B(\vec{r}_0+\vec{r})\rangle$$

Furthermore, as the chromo-magnetokinetic term also generates rotations, we need to construct states that are irreducible with respect to the rotation group of the cube. This prescription insures that when we do eventually calculate mass ratios, the glueballs derived from the original boxiton states of our lattice will be eigenstates of angular momentum characterized by a definite J^{π} assignment.

In the y=0 limit of Hamiltonian strong coupling expansion, our scalar, axial vector and tensor boxiton states of zero momentum :

$\sqrt{6}\,|S\rangle =$ 

$\sqrt{2}\,|AV\rangle =$ 

$2\,|T\rangle =$ 

are clearly degenerate. Perturbative effects will now split this degeneracy. We consider the time propagation of a static boxiton, a situation which is easily visualized by picturing a single plaquette, oriented with its normal along a "time axis", and the surface which is swept out by its links as time evolves. Thus, for example, we see in fig.4 the propagation of a boxiton that is created by the Wilson trace operator $Tr\{U(1)U(2)U(3)U(4)\}$ at time t_1 and is subsequently destroyed at a later time t_2 by the operator $\mathrm{Tr}\{U^\dagger(4)U^\dagger(3)U^\dagger(2)U^\dagger(1)\}$.

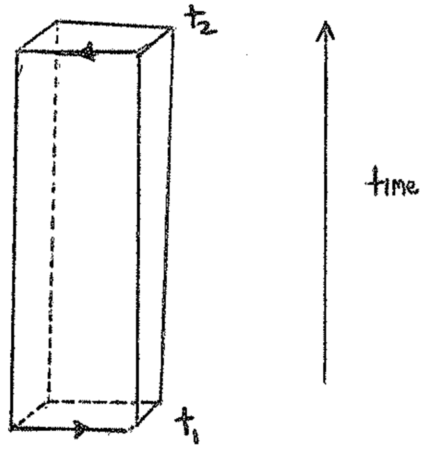


fig. 4.

To compute the interesting higher order effects, we apply Rayleigh-Schrodinger perturbation to our Wamiltonian (✱). If we let $P_0 = |i\rangle\langle i|$ be the projection operator for the initial unperturbed state (n.b. here i=S,AV,T) with associated unperturbed energy ω_0, we obtain the following series for the Wamiltonian energy eigenvalue ω_i:

$$
\begin{aligned}
\omega_i = \omega_0 &+ y\langle i|W_{kinetic}|i\rangle \\
&+ y^2\langle i|W_{kin}\frac{1-P_0}{\omega_0-W_0}W_{kin}|i\rangle \\
&+ y^3\left\{\langle i|W_{kin}\frac{1-P_0}{\omega_0-W_0}W_{kin}\frac{1-P_0}{\omega_0-W_0}W_{kin}|i\rangle - \langle i|W_{kin}\frac{1-P_0}{(\omega_0-W_0)^2}W_{kin}P_0W_{kin}|i\rangle\right\} \\
&+ \ldots
\end{aligned}
$$

Since the action of the perturbative piece of the Wamiltonian is merely to create/destroy boxitons, it is a simple matter to associate diagrams with each of the terms in the above strong coupling expansion. Each factor of $W_{kinetic}$ is represented pictorially by the appearance of an oriented plaquette, while the intervening factors of $\frac{1-P_0}{\omega_0-W_0}$ permit the oriented plaquette to propagate with time, its string bits sweeping out a parallelipiped surface, until the next perturbation factor alters the character of the intermediate state. Diagrams contributing up to y^2 are shown below:

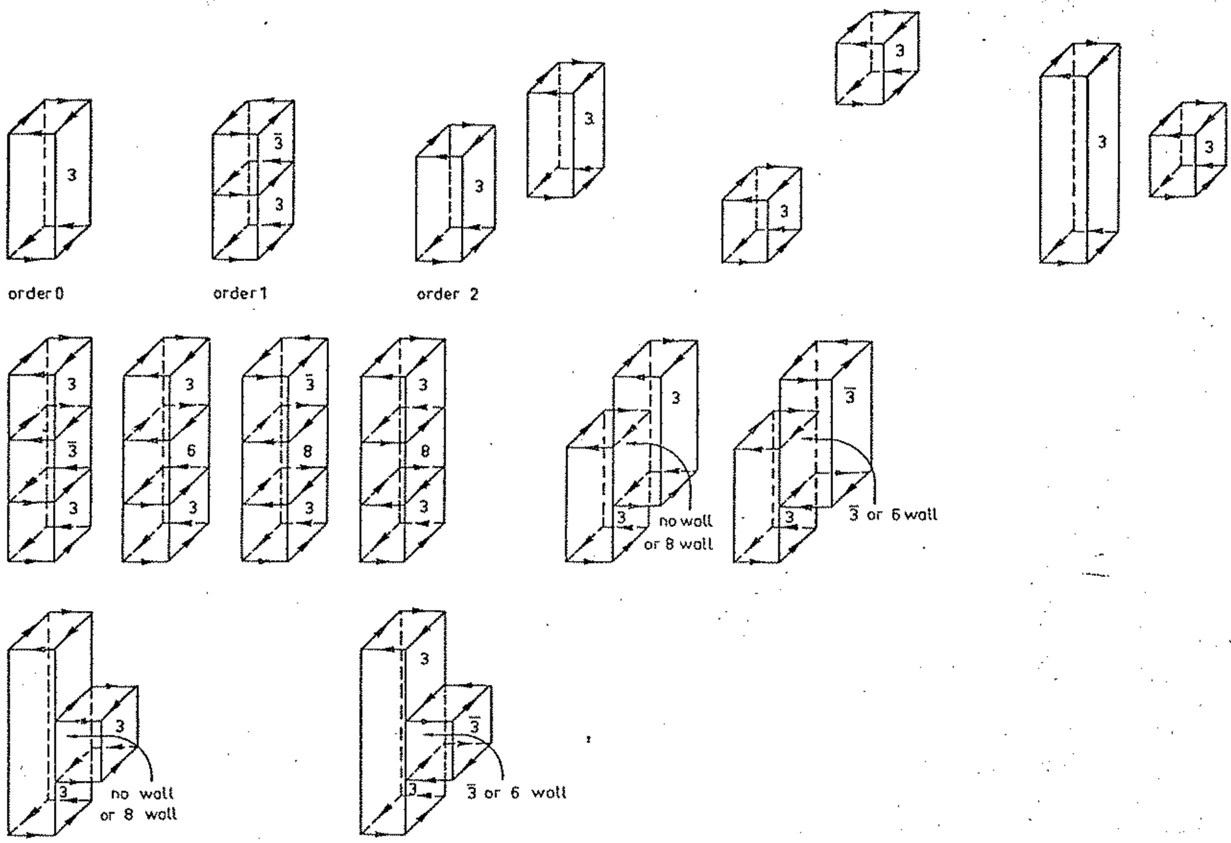


Fig. 12. Graphs contributing up to second order to the energy level of the boxciton.

Kogut, Sinclair and Susskind, after many hours of intense mental labor (Robson contends that the calculational time scale of this method is measured not in days or weeks, but in years!), obtained the following expansions, up to fourth order, for the scalar, axial vector, and tensor boxiton energies:

$$\omega_S = \frac{16}{3}\left(1 - \bar{y} - .569\,\bar{y}^2 + 17.393\,\bar{y}^3 - 95.206\,\bar{y}^4\right)$$

$$\omega_{AV} = \frac{16}{3}\left(1 + \bar{y} + .097\,\bar{y}^2 - 3.112\,\bar{y}^3 - 51.840\,\bar{y}^4\right) \qquad \bar{y} \equiv \frac{y}{(16/3)}$$

$$\omega_T = \frac{16}{3}\left(1 - \bar{y} + .583\,\bar{y}^2 - 7.650\,\bar{y}^3 - 15.357\,\bar{y}^4\right)$$

Because asymptotic freedom dictates that the non-Abelian lattice gauge theory coupling constant g approach zero in a well prescribed fashion as the lattice spacing vanishes, Kogut, Sinclair and Susskind could extrapolate the above expansions to the continuum limit using Padé techniques. As $y \to \infty$, they discovered the glueball mass ratios:

$$\frac{M_{AV}}{M_S} = 1.575 \qquad \frac{M_T}{M_S} = 1.003$$

e. Mass Gap in the Euclidean Formulation of the Lattice Theory

In the Euclidean version of the lattice gauge theory, invented independently by Polyakov[23] and Wilson[24], one discretizes both space and imaginary time, thereby regularizing the original quantum field theory and relegating the real world continuum to the status of a statistical mechanical system on a four dimensional hypercubic lattice of infinite extent. Here we concentrate on the theoretical similarities of statistical mechanics in four dimensions and quantum field theory in 3+1 dimensions in preparation for a discussion of the Euclidean lattice gauge theory strong coupling expansion for the mass gap. In addition to such a high temperature expansion, statistical mechanics provides the lattice gauge theorist with other powerful investigatory tools such as Monte Carlo methods, rigorous inequalities and, in some cases, low temperature expansions as well. At the heart of this connection between statistical mechanics and quantum field theory one finds the transfer matrix, invented years ago by Schultz, Mattis and Lieb[25] in their studies of the Ising spin system in two dimensions. In Minkowski space quantum mechanics the time evolution operator is $e^{\frac{i}{\hbar}Ht}$. In Euclidean space of imaginary time the corresponding operator is $e^{-H_E\tau}$. For Euclidean lattice gauge theories, however, it is the real, symmetric transfer matrix $\hat{T}$ which propagates the fields of the theory in the temporal direction. In the ensuing discussion, following Kogut[26], we consider only simple scalar fields $\varphi(n)$ defined on the sites of the lattice. One can define the Hamiltonian of the lattice theory via the relation

$$H_s = -\frac{1}{\tau} \ln \hat{T}$$

though for our Euclidean treatment of the lattice the resulting expression for H_s is none too enlightening. In addition, a celebrated result allows one to write the partition function of the theory

$$Z = \prod_n \int_{-\infty}^{\infty} d\varphi(n)\, e^{-S}$$

as

$$Z = \mathrm{Tr}\, \hat{T}^{N+1}$$

where N is the number of "temporal" planes in the lattice. We now proceed to present some interesting correspondences that exist between the statistical mechanical and quantum field theoretic visions of the lattice system. In the latter, one deals with the field $\hat{\varphi}(n)$, its conjugate momenta $\hat{\pi}(n)$, and the Hamiltonian H_S defined on the symmetric lattice, whilst in the former the partition function Z provides a complete description of the system.

Traditionally, a la Feynman, the propagator is defined as the vacuum expectation value of the time ordered product of fields φ in the Heisenberg representation

$$\Delta(t,\vec{x}) = \langle 0| T\{\hat{\varphi}(t,\vec{x})\hat{\varphi}(0,0)\}|0\rangle$$

where $|0\rangle$ is the ground state of our Hamiltonian H_S. Switching to the Schrodinger representation and choosing $t>0$, we get

$$\Delta(t,\vec{x}) = \langle 0|\hat{\varphi}(\vec{x})\, e^{-iH_s t}\, \hat{\varphi}(0)|0\rangle\, e^{iE_0 t} \qquad (*)$$

where E_0 is the energy of the quantum vacuum. By contrast, if one considers the correlation function

$$\Gamma(n_0,\vec{n}) = \frac{1}{Z}\int \prod_{n_0',\vec{n}'} d\varphi(n_0',\vec{n}')\,\varphi(n_0,\vec{n})\varphi(0,\vec{0})\,e^{-S}$$

and organizes the integrals so that they are performed successively over the temporal planes of the lattice, only for the slices $n_0'=0, n_0$ will the calculations differ from those for Z below. Since the transfer matrix connects adjacent slices, we will get the expression

$$\Gamma(n_0,\vec{n}) = \frac{\mathrm{Tr}\left\{\hat{\varphi}(\vec{n})\,\hat{T}^{n_0}\,\hat{\varphi}(0)\,\hat{T}^{N+1-n_0}\right\}}{\mathrm{Tr}\,\hat{T}^{N+1}}$$

Using the transfer matrix definition of the Hamiltonian, as well as the important result $\hat{T}^{N+1} \xrightarrow[N]{\infty} |0\rangle e^{-E_0\tau}\langle 0|$, we obtain

$$\Gamma(n_0,\vec{n}) = \langle 0|\,\hat{\varphi}(\vec{n})\,e^{-n_0 H_S \tau}\,\hat{\varphi}(0)\,|0\rangle\,e^{n_0 E_0 \tau}$$

Comparison with (*) reveals the correspondence

$$\Gamma(n_0,\vec{n}) = \Delta(-i n_0\tau,\vec{n})$$

so that knowledge of the statistical mechanics correlation function informs us about the field theoretic propagator for discrete values of imaginary time.

Lastly, we show that the mass gap of the lattice field theory is intimately related to the inverse correlation length of the associated statistical mechanical system. It is a well known fact of condensed matter physics that for a system not critical, the correlation function falls exponentially as

$$\Gamma(n_0,0) \sim \exp\left(-|n_0|/\xi\right)$$

for $|n_0| \gg \xi$, where ξ is the aforementioned correlation length. Motivated by the previously established connection, we insert a complete set of eigenstates, $1=\sum_{\ell}|\ell\rangle\langle\ell|$, of the Hamiltonian H_S into the propagator $\Delta(-i n_0\tau, 0)$ to obtain the equation,

$$\Delta(-in_0\tau,0) = \sum_{\ell} e^{-(E_\ell - E_0)n_0\tau} |\langle 0|\hat{\varphi}(0)|\ell\rangle|^2$$

for $n_0\tau \gg 1$, only the particular eigenstate ℓ minimizing $(E_\ell - E_0)$ will not be exponentially lost. Since the lowest nonzero $(E_\ell - E_0)$ value in the system is just the mass of the lightest particle of the theory at zero momentum, we have, in this limit,

$$\Delta(-in_0\tau,0) \sim \exp(-mn_0\tau)$$

so that

$$ma = \frac{1}{\xi}$$

in our Euclidean version where the discrete time interval τ is just the lattice spacing a.

Having just established this connection between the mass gap of the lattice gauge theory and the correlation length of the statistical mechanical system, we next present the strong coupling cluster expansion technique of Munster[7] in order that we might study the exponential decay of the plaquette-plaquette correlation function in the three dimensional Euclidean theory.

The partition function of the lattice gauge theory system is given by the integral

$$Z = \int \prod_b dU(b)\, e^{L} \qquad (\blacksquare)$$

where the gauge field variables $U(b) \in SU(2)$ are attached to the links b of the lattice, dU is the Haar measure of the group, and L is the action of the pure lattice theory

$$L = \sum_{\text{plaquettes}} L_p = \sum_{\text{plaquettes}} \beta d_\chi^{-1}\, \chi[U(\dot{p})]$$

with $U(\dot{p})$ being the directed product of link variables around the plaquette, $d_\chi \equiv \chi(\mathbb{1})$ the dimension of a faithful representation of the gauge group, and χ the real part of its character. Thanks to the Peter-Weyl theorem, one can write down a Fourier expansion for e^{L_p} in terms of the unitary irreps of the compact Lie group G of interest

$$\exp L_p = \sum_{\nu \in \hat{G}} c_\nu(\beta)\, \chi_\nu[U(\dot{p})] \qquad (**)$$

where $\hat{G}$ is the set of all such inequivalent irreps. Utilization of the generalized orthogonality relation between characters

$$\int dU\, \chi_r(U)\, \chi_s(U^{-1}) = \delta_{rs}$$

permits one to determine the Fourier coefficients via the formula

$$C_\nu(\beta) = \int_G dU\, \chi_\nu(U^{-1}) \exp L_p$$

For the gauge group SU(2), with plaquette action $L_p = \frac{1}{2}\,\mathrm{tr}\{U(\dot{p})\}$, one can, with some labor, obtain the following expression for the coefficients of the Peter-Weyl expansion

$$C_j(\beta) = \frac{2(2j+1)}{\beta}\, I_{2j+1}(\beta)$$

where the subscript j refers to the (2j+1)-dimensional irrep of the group and I_n is the modified Bessel function. Typically, one writes (**) as

$$\exp L_p = c_0(\beta)\left\{1 + \sum_{j\neq 0} \frac{c_j(\beta)}{c_0(\beta)}\, \chi_j[U(\dot{p})]\right\}$$

then drops the $c_0(\beta)$ factor outside the brackets since it would, in any case, be canceled later on. Hence we have

$$\exp L_p = 1 + \sum_{j\neq 0} (2j+1)\frac{I_{2j+1}(\beta)}{I_1(\beta)}\chi_j[U(\dot{p})] \equiv 1 + \sum_{j\neq 0} d_j a_j(\beta)\chi_j \equiv 1 + f_p$$

so that we are able to write an expansion for the exponentiated lattice action

$$\exp L = \exp\Big(\sum_p L_p\Big) = \prod_p \exp L_p = \prod_p (1+f_p)$$

which yields the crucial result

$$e^L = \sum_{\mathcal{P}} \prod_{p\in\mathcal{P}} f_p \qquad (***)$$

where the sum extends over all possible sets $\mathcal{P} = \{P_1, P_2, \ldots\}$ of plaquettes in our lattice. Using (■) and (■■■), we can now write the partition function as

$$Z = \sum_{\mathcal{P}} \int \prod_b dU(b) \prod_{p \in \mathcal{P}} f_p$$

Decomposing an arbitrary set $\mathcal{P}$ into connected components X_i, which are finite sets of linkwise connected plaquettes, we write $\mathcal{P} = \sum_i X_i$ and note that because the integrals performed in the calculation of Z are over the link variables, the contributions of the disjoint components X_i factorize into distinct products

$$\Phi(X_i) \equiv \int \prod_b dU(b) \prod_{p \in X_i} f_p$$

so that

$$Z = \sum_{\mathcal{P} = \sum X_i} \prod_i \Phi(X_i)$$

Recalling the relation $\int dU\, \chi_r(U) = \delta_{r0}$ and the fact that the power factor f_p <u>does not</u> contain the trivial j=0 irrep, it is clear that $\Phi(X_i)$, the "activity" of the component X_i, will vanish if X_i contains a free link in the interior of the lattice. Thus it is apparent that the only connected sets, X_i, of plaquettes that will contribute a nonzero activity to the partition function will be those which form closed surfaces. Munster calls such connected sets "polymers", since he has, via the above procedure, effected a transformation of the pure lattice gauge theory into a Gruber-Kunz[27] polymer system with partition function

$$Z = \sum_{\mathcal{D}} \Phi(\mathcal{D}) = \sum_{\mathcal{D}} \prod_{X \in \mathcal{D}} \Phi(X)$$

where the sum runs over all sets, $\mathcal{D}$, of mutually disconnected polymers.

As objects of interest in lattice gauge theories generally involve the logarithm of the partition function rather than Z itself, it will be necessary for us to construct a related expansion for ln Z, in terms of terms of connected, rather than the disconnected graphs as above. For translationally invariant systems, one could examine, traditionally, the occurrence factors of particular graphs in the lattice and determine the expansion for ln Z by merely "picking off" the coefficients of the terms in Z linear in the lattice volume. However, in the computation of the mass gap, as in Munster's calculation for the string tension, one deals with polymers that will stretch the full length of the lattice so that an alternative approach is necessary to obtain an expansion for ln Z. The needed tool employed by Munster in his original work was the moment-cumulant transformation, which allowed him to derive the famous "cluster" expansion,

$$\ln Z = \sum_{C} a(C) \prod_{X_i \in C} \Phi(X_i)^{n_i}$$

where a cluster is defined to be a connected set of different polymers X_i, $i=1,\ldots,k$, with multiplicities n_i, and where the sum itself runs over all permissible clusters. For convenience, one summarizes the structure of a cluster via the notation $C=(X_1^{n_1},\ldots,X_k^{n_k})$. The combinatorical coefficients a(C) are dependent upon the cumulants of the theory

$$a(C) = \frac{1}{\prod_i n_i!}\Big[\overbrace{X_1,\ldots\ldots,X_1}^{n_1};\ldots\ldots;\overbrace{X_k,\ldots\ldots,X_k}^{n_k}\Big] \equiv \frac{1}{\prod_i n_i!}\,\hat{a}(C)$$

while calculation of the cumulants is accomplished by means of the formula

$$[\alpha,\ldots\ldots,\delta] = \sum_{\text{PERM}} (-1)^{k-1}(k-1)!\,\overbrace{\langle\alpha,\ldots\ldots,\beta\rangle\langle\gamma,\ldots\ldots,\delta\rangle\ldots\ldots\langle\mu,\ldots,\nu\rangle}^{k\text{ factors}}$$

and the initial prescription for the moments

$$\langle X_1,\ldots\ldots,X_k\rangle = \begin{cases} 1, & \text{if every pair } X_i, X_j \text{ is not connected} \\ 0, & \text{otherwise}\end{cases}$$

$$\langle\phi\rangle = 0$$

Because the cumulant $[X_1, \ldots, X_k]=0$ unless $X_1 \cup \ldots \cup X_k$ is connected, it is clear that Munster's cluster expansion for ln Z involves consideration of connected graphs only.

To calculate the mass gap of the three dimensional Euclidean lattice theory, we consider the correlation of two spacelike plaquettes separated by a time distance t:

$$\rho(t) = \langle \chi[U_{p_2}(t)]\chi[U_{p_1}(0)]\rangle - \langle \chi[U_{p_2}(t)]\rangle\langle \chi[U_{p_1}(0)]\rangle \qquad (\Delta)$$

From our previous discussion we know that the mass gap will be given by

$$m = -\lim_{t\to\infty} \frac{1}{t} \ln \rho(t)$$

Munster[7] conveniently expresses the plaquette-plaquette correlation function in terms of the cluster expansion for ln Z by considering couplings β_1, β_2 on the two plaquettes of interest and standard coupling β on all other plaquettes

$$\rho(t) = (\mathrm{tr}\,\mathbb{1})^2 \frac{\partial^2}{\partial\beta_1 \partial\beta_2} \ln Z(\beta_1, \beta_2, \beta)\Big|_{\beta_i=\beta} \qquad (\Delta\Delta)$$

A quick check reveals that the above prescription reproduces our definition of the plaquette-plaquette correlation. It is apparent from the presence of the two differential operators that all clusters contributing to $\rho(t)$ will necessarily contain the two plaquettes P_1 and P_2 under scrutiny. To compute the first order approximation to the mass gap, we note that the minimal cluster X_0 contributing to the expansion is a rectangular box with P_1, P_2 as the end plaquettes and the walls of the cluster composed entirely of plaquettes in the fundamental ($j=\frac{1}{2}$) representation of SU(2) (cf. fig. 4′).

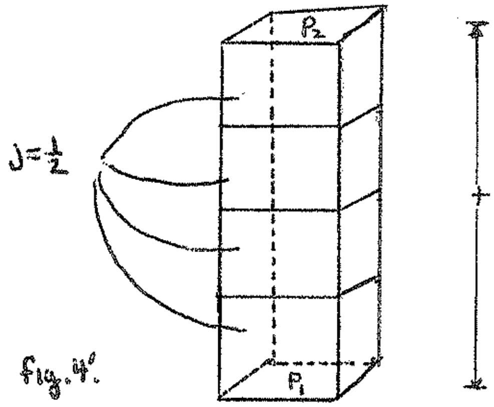


fig. 4c.

Performing the integration over links using the generalized orthogonality relation for characters

$$\int dU\,\chi(US)\,\chi(U^{-1}T) = d_\chi^{-1}\,\chi(ST)$$

one obtains the following activity for the lowest order polymer

$$\Phi(X_0) = (d_f)^2 a_f(\beta_1) a_f(\beta_2)\left[a_f(\beta)\right]^{4t} \equiv (d_f)^2 U(\beta_1) U(\beta_2)\left[U(\beta)\right]^{4t}$$

since there are 4t plaquettes along the side walls for a P_1, P_2 separation of time t. Substituting this result into (A6), one gets

$$\rho_0(t) = \left(d_f^2 \frac{dU}{d\beta}\right)^2 \left[U(\beta)\right]^{4t} = A e^{-m_0 t}$$

with

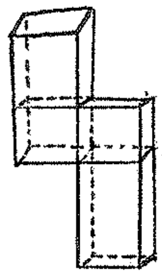
$$m_0 = -4 \ln U$$

where we've made use of the fact that, for large coupling (small β), $a_f(\beta) = \frac{I_2(\beta)}{I_1(\beta)} \sim \beta$ so that $\frac{dU}{d\beta} \sim$ constant. Higher order corrections to m_0 will come from clusters of increasing numbers of plaquettes, which result in more powers of β. A recent revelation by Munster has revealed that in order for one to obtain an expansion that exponentiates in the form

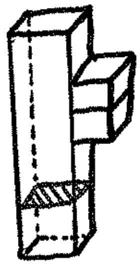
$$\exp(-mt) = \exp(-m_0 t)\exp(-\Delta m t) = U^{4t}\left\{1 - \Delta m t + \tfrac{1}{2}(\Delta m)^2 t^2 - \ldots\right\}$$

it is necessary to consider not position eigenstate plaquettes, but rather zero momentum eigenstates. This requirement is incorporated into the cluster expansion technique by summing over all spacelike orientations of P_1 and P_2 separately and insures us that we will really be computing $E(\vec{p}=0)=m_g$ without unknowingly adding the kinetic energy contributions associated with the position eigenstate.

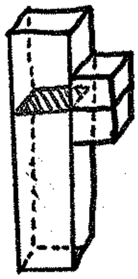
Formulation of a direct cluster expansion for $\ln \rho(t)$ is a straightforward matter. One merely recasts the cluster expansion for $\rho(t)$, which, by virtue of Munster's revelation, now contains terms such as



into the guise of a moment expansion, by envisioning, in an abstract sense, all higher order graphs to be built from either the lowest order polymer X_0 or its essential variants (e.g.  ,for the above). Next, redefining what one means by a "polymer" so that a factor of $\Phi(X_0)$, the activity of the minimal cluster, can be pulled out of the expansion, it is possible, via a second moment-cumulant transformation, to obtain a series for $\ln \rho(t)$ in terms of connected graphs only. The existence of this direct cluster expansion for $\ln \rho(t)$ implies that $\rho(t)$ itself, now written as a moment expansion, exponentiates as desired. Since this is true, we can quickly write down the expansion for $\ln \rho(t)$ by merely picking off the coefficients of the terms in $\rho(t)$ linear in t. These considerations allow us to determine Δm, the correction to m_0. For example, we note that for the polymer



the number of possible arrangements is $4t(t-1)$ since the double cube decoration can sit on any of the four walls of the minimal cluster, while the $j=1$ irrep plaquette has $t-1$ available positions. (Ordinarily, there are t possible ways to insert a single plaquette into X_0, but for the above polymer there is one less since the configuration



has vanishing activity due to the free link of the $j=1$ plaquette.) The term proportional to t, the plaquette-plaquette separation, is $\kappa=-4$. With the activity $\Phi(\Xi) = \Phi(X_0)[a_{\frac{1}{2}}(\beta)]^6[3a_1(\beta)] \equiv \Phi(X_0)\hat{\Phi}(\Xi)$,we get the contribution $-4[a_{\frac{1}{2}}(\beta)]^6[3a_1(\beta)]$ to $-\Delta m$. (n.b. the uncanceled factor of $d_1=3$ prefixing $a_1(\beta)$ is a consequence of the fact that the insert functions as a passive plaquette in this polymer.) In the appendix to this section we list all graphs contributing to Δm up to 8^{th} order in β. Writing $u=a_{\frac{1}{2}}$, $v=a_1$, $w=a_{\frac{3}{2}}$, we obtain the following strong coupling expansion for the mass gap of the three dimensional Euclidean lattice gauge theory:

$$\begin{aligned} m = & -4\ln u - \ln(1+3v-4u^2) - 8u^4 - 24u^4v + 32u^6 \\ & -4u^{-2}v^4 + 48u^{-2}v^5 - 16u^{-3}v^4w - 32v^4 \\ & - \quad u^8 + \quad u^6v \end{aligned}$$

Appendix: Graphs in the strong coupling expansion of m_g.

On the next few pages we tabulate the graphs which contribute to the mass gap of the SU(2) lattice gauge theory in 3 dimensions.

First, the polymers consisting of the minimal cluster X_0 plus plaquette insertions are listed since they can be summed separately and lead ultimately to the lattice gas term, $\ln(1+3v-4u^2)$, in our strong coupling expansion. The table gives the activities $\hat{\Phi}(\Xi_\lambda) = \Phi(\Xi_\lambda)\cdot\Phi(X_0)^{-1}$ in terms of u, v, and w, associated with the higher order effect. Lastly, the counting factor, κ, is also listed. After the lattice gas contributions follow the standard clusters. We break them up into distinct groups depending upon the value of the combinatorical coefficient a(C).

Comparison with Munster's recently released results[7] reveals a discrepancy in the last $8^{\underline{th}}$ order term .

Finally, we note that Munster's work in four dimensions permitted him to gain an estimate for the mass gap-string tension ratio of the SU(2) lattice theory,

$$\frac{M_g}{\sqrt{\sigma}} = 1.8 \mp 0.8$$

that was lower than all values previously established using Monte Carlo methods. This result was expected, however, since Munster considered true momentum zero eigenstates, whilst the Monte Carlo people failed to do so.

IV. Monte Carlo Work

a. 3-Dimensional QCD

Motivated by the fact that quantum chromodynamics in three dimensions lacks the instanton solutions which Callan, Dashen and Gross[6] contend will fill the β-function gap seen in the sudden crossover from strong to weak coupling in the four dimensional theory, D'Hoker[28] employed Creutz' Monte Carlo methods[5] to study the three dimensional Euclidean lattice theory. In addition to the string tension and the internal energy, D'Hoker investigated the temperature dependence of the mass gap of the lattice field theory, which not only provided him with a check on the phase diagram of the system, but also permitted him to ascertain the important ratio $\frac{M}{\sqrt{\sigma}}$ in the continuum limit.

Our earlier discussion of the connections between four dimensional statistical mechanical systems and quantum field theories in 3+1 dimensions dictated that the mass gap was nothing more than the inverse correlation length. Hence, in the Monte Carlo calculation of the mass gap, we consider the correlation between coaxial plaquettes located a distance d=na apart:

$$\mathcal{E}_j(n) = \langle \Box_j(n) \Box_j(0) \rangle - \langle \Box_j \rangle^2$$

where $\Box_j(n) = \mathrm{Tr}\left[U_j(P(n))\right]$ is the Wilson trace associated with the plaquette at position n for the $j^{\underline{th}}$ iteration and where we average over <u>all</u> such pairs of plaquettes in D'Hoker's 27^3 lattice system. Next, averaging over all but the first five iterations or so, which the system needs to reach an "equilibrium", we can obtain the Monte Carlo estimate for the plaquette-plaquette correlation,

$$\rho(n) = \frac{1}{J-5} \sum_{j=6}^{J} \mathcal{E}_j(n)$$

J= total number of iterations

Recalling that the mass gap Ma is defined via the decay of the correlation function,

$$\rho(n) \underset{n}{\overset{\infty}{\propto}} \exp(-naM)$$

we can, in principle, easily estimate its value from the Monte Carlo results obtained for $\rho(n)$,

$$Ma = \ln\left[\frac{\rho(n)}{\rho(n+1)}\right]$$

Making runs of 25 iterations for β between 2.0 and 5.0, D'Hoker discovered that typically the standard deviations of $\rho(n)$ would overlap with zero for $n \geqslant 3$ so that there was little chance to establish the expected exponential decay of the correlation function. Consequently it was necessary for D'Hoker to assume the exponential behavior for $\rho(n)$ and he was forced to compute the mass gap from just $\rho(1)$ and $\rho(2)$. Plotted in fig. 5 is the β-dependence of the mass gap in 3-dimensional QCD as revealed by a weak coupling fit to the Monte Carlo data. In addition, the first order term in Munster's strong coupling expansion is included to allow comparison with the computer generated numbers in this regime. As is apparent from the graph itself, the mass gap does not vanish for finite β, thereby precluding the possibility of a second order transition at nonzero coupling. Using his string tension data, D'Hoker was also able to plot $\frac{M}{\sqrt{\sigma}}$ as a function of β, which is of interest because one expects that as the lattice spacing goes to zero, the ratio $\frac{M}{\sqrt{\sigma}}$, as it represents a real, physical quantity, should level off at some finite value (cf. fig. 6). In the continuum limit, D'Hoker determined the ratio to be

$$\frac{M}{\sqrt{\sigma}} = 4.5 \pm 0.5$$

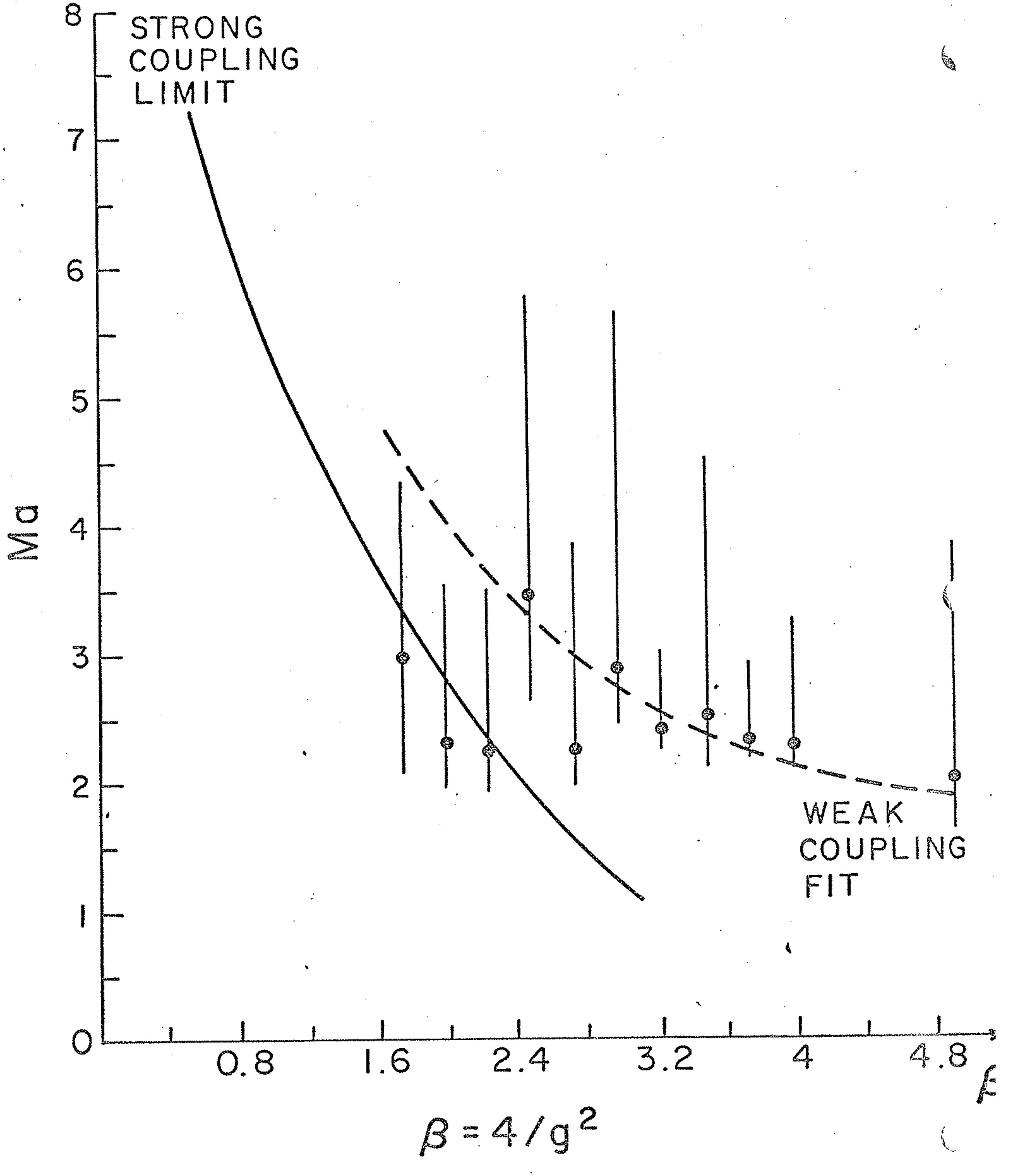

STRONG
COUPLING
LIMIT
Ma
8
7
6
5
4
3
2
1
0
0.8
1.6
2.4
3.2
4
4.8
WEAK
COUPLING
FIT
$\beta = 4/g^2$

FIG.5

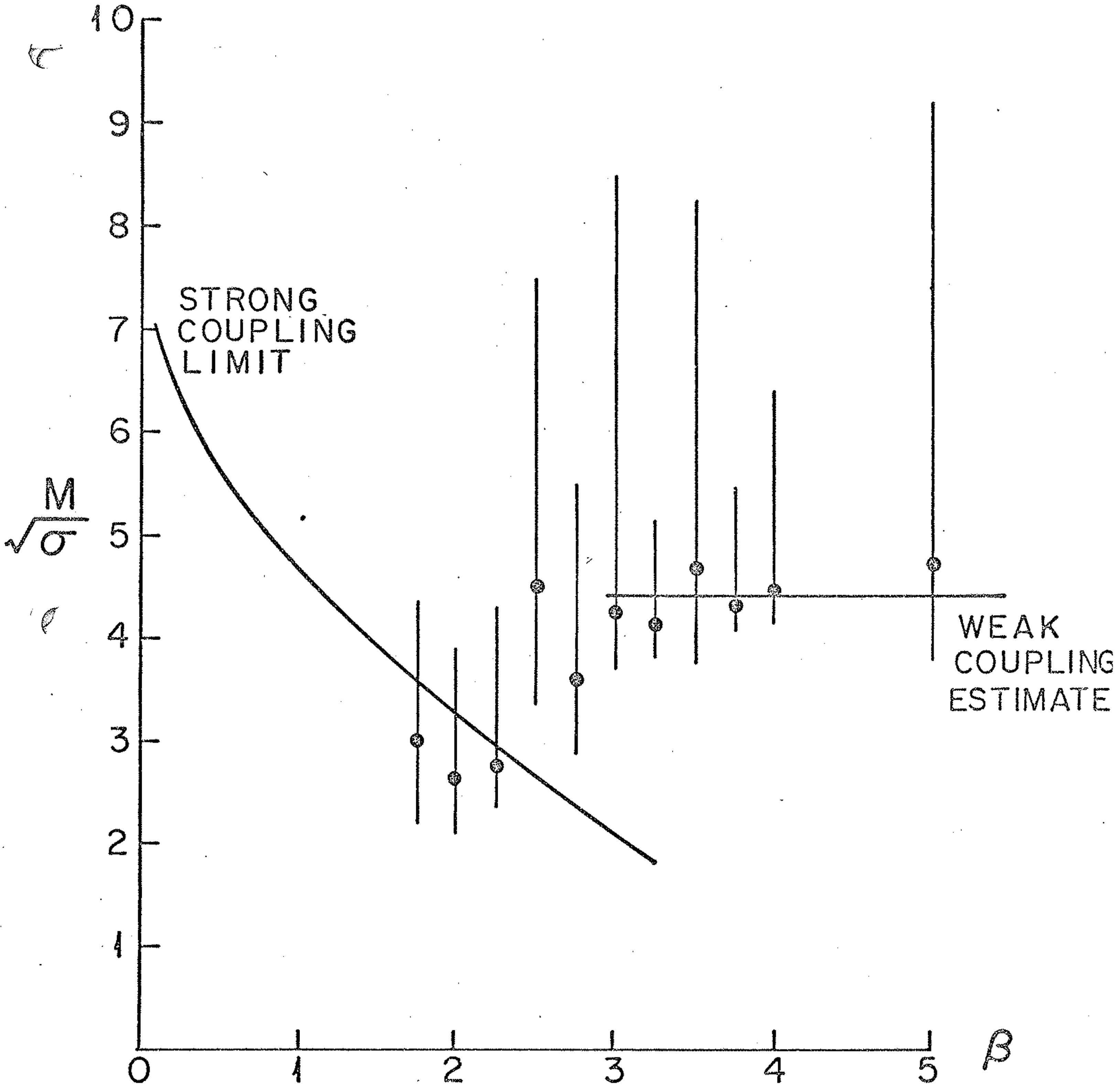

10
9
8
7
6
5
4
3
2
1
$\frac{M}{\sqrt{\sigma}}$
0
1
2
3
4
5
β
STRONG
COUPLING
LIMIT
WEAK
COUPLING
ESTIMATE

FIG. 6

b. 4-Dimensional QCD

In the first attempt to measure the mass gap of the four dimensional theory using Monte Carlo methods, Berg[29] studied the behavior of the plaquette-plaquette correlation function on 6^4 and 9^4 lattices. Making as many as 16045 iterations per run on the smaller lattice and 8482 on the larger, Berg, nevertheless, still had trouble resurrecting the standard deviations of his numbers from the zero line (cf. fig. 7). For all distances $n \geq 3$ the value of the correlation function itself was less than its standard deviation and even the results for $\rho(2)$ were of borderline credibility- all of Berg's measurements being the victim not only of the statistical fluctuations associated with the Monte Carlo process, but also of the finite size effects of his small lattices. Because of the poor quality of his correlation function data, there was little hope that Berg could obtain the mass gap directly.

fig. 7.

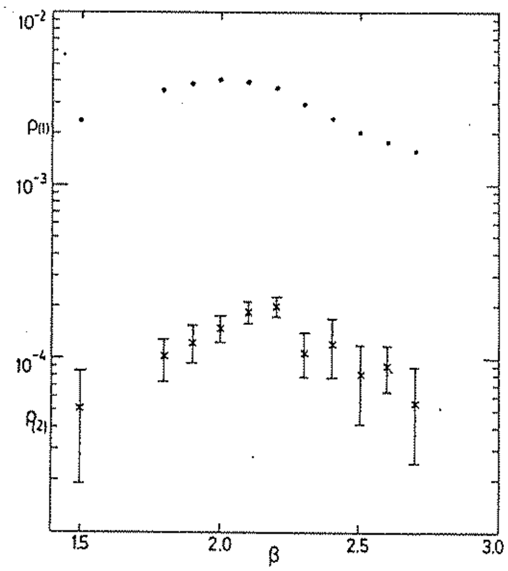


Fig. 7. Data for $\rho(n)$ ($n = 1, 2$). The error bars for $\rho(1)$ are negligible on the used scale).

Instead, he notes that since the lattice theory defines a continuum quantum field theory as the lattice spacing vanishes, asymptotic freedom dictates that the correlation length be dependent upon β in the following manner-

$$\xi(\beta) \sim \beta^{-\frac{51}{121}} \exp\left(\frac{3}{11}\pi^2\beta\right)$$

The above formula provides Berg with an upper bound for ξ, which corresponds to the lower bound $\frac{M}{\sqrt{\sigma}} > 1.8$, for the mass gap-string tension ratio, when his reliable string tension data are employed. To actually salvage an absolute estimate for the ratio $\frac{M}{\sqrt{\sigma}}$, Berg plots his Monte Carlo points $\left(q(\beta) = \frac{\rho(2)}{\rho(1)} \Rightarrow -\frac{1}{\ln q(\beta)} = \xi\right)$, the very close fitting first order strong coupling term, and the aforementioned upper bound prescribed by asymptotic freedom (cf. fig. 8).

fig. 8.

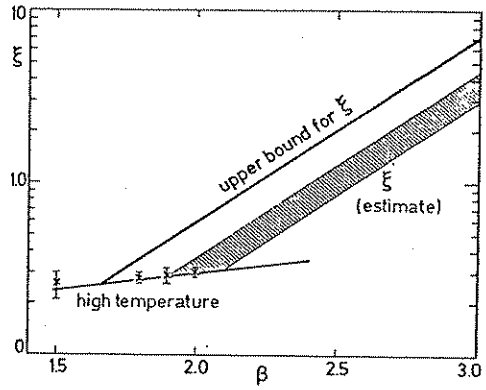


Fig. 8. Correlation length: Lowest order high temperature expansion and Monte Carlo points for $-1/\ln q(\beta)$ up to $\beta = 2.0$; upper bound for large β and estimate relying on the present Monte Carlo data for large β.

Noting that his Monte Carlo estimate for the correlation length at $\beta=2.0$ still remained on the straight line of the lowest order high temperature expansion, but that his plot of the string tension revealed the onset of asymptotic weak coupling behavior at $\beta=2.1$, Berg assumed the same for ξ somewhere in the range $1.9 \leq \beta \leq 2.1$. Such reasoning led him to the result

$$\frac{M}{\sqrt{\sigma}} = 3.7 \mp 1.2$$

Late in 1980, Bhanot and Rebbi[30], two physicists at BNL who were aware, perhaps, of the poor statistics associated with Berg's small lattices, performed a mass gap Monte Carlo calculation using not the standard continuous gauge group SU(2), but rather $\tilde{Y}$, its 120-element non-Abelian icosahedron subgroup. Utilization of this finite subgroup drastically reduced the processing time and memory requirements of the corresponding Monte Carlo computer program so that Bhanot and Rebbi could employ very large, unprecedented 16^4 lattices. Previous work by Rebbi[31], as well as Petcher and Weingartan[32], had established the validity of the method and revealed that there were no problems in using the finite subgroup provided the characteristic transition to the broken symmetry phase occurred at a β outside the region of interest. Since $\beta_c \gtrsim 5.99$ for $\tilde{Y}$ and there was little chance of performing Monte Carlo work for $\beta \geqslant 5.0$ to begin with, Bhanot and Rebbi found that the finite subgroup worked quite well for their purposes. The use of a large lattice by the physicists led to good statistics and a minimization of finite size effects. Furthermore, because they had a very large CDC 7600 computer at their disposal, Bhanot and Rebbi were able to grind through the 16^4 lattice in a mere 15 seconds/iteration to obtain estimates for the dimensionless mass gap as a function of β,

$$\mu(d,\beta) = \ln\left[\frac{\rho(d)}{\rho(d+1)}\right]$$

for distances $d \leqslant 4$ (cf. fig. 9). Plaquette-plaquette correlations for spacings greater than four were buried beneath the noise of Monte Carlo statistical fluctuations. Although, in a strict sense, one only obtains the physical glueball mass in the limit,

$$M = \lim_{\substack{a \to 0 \\ d \to \infty}} \frac{1}{a} \mu(d,\beta)$$

it is true that for large enough β, $\mu(d,\beta)$ will behave perturbatively for any finite d, so that the curve which determines the glueball mass will be the envelope of the mass gap curves.

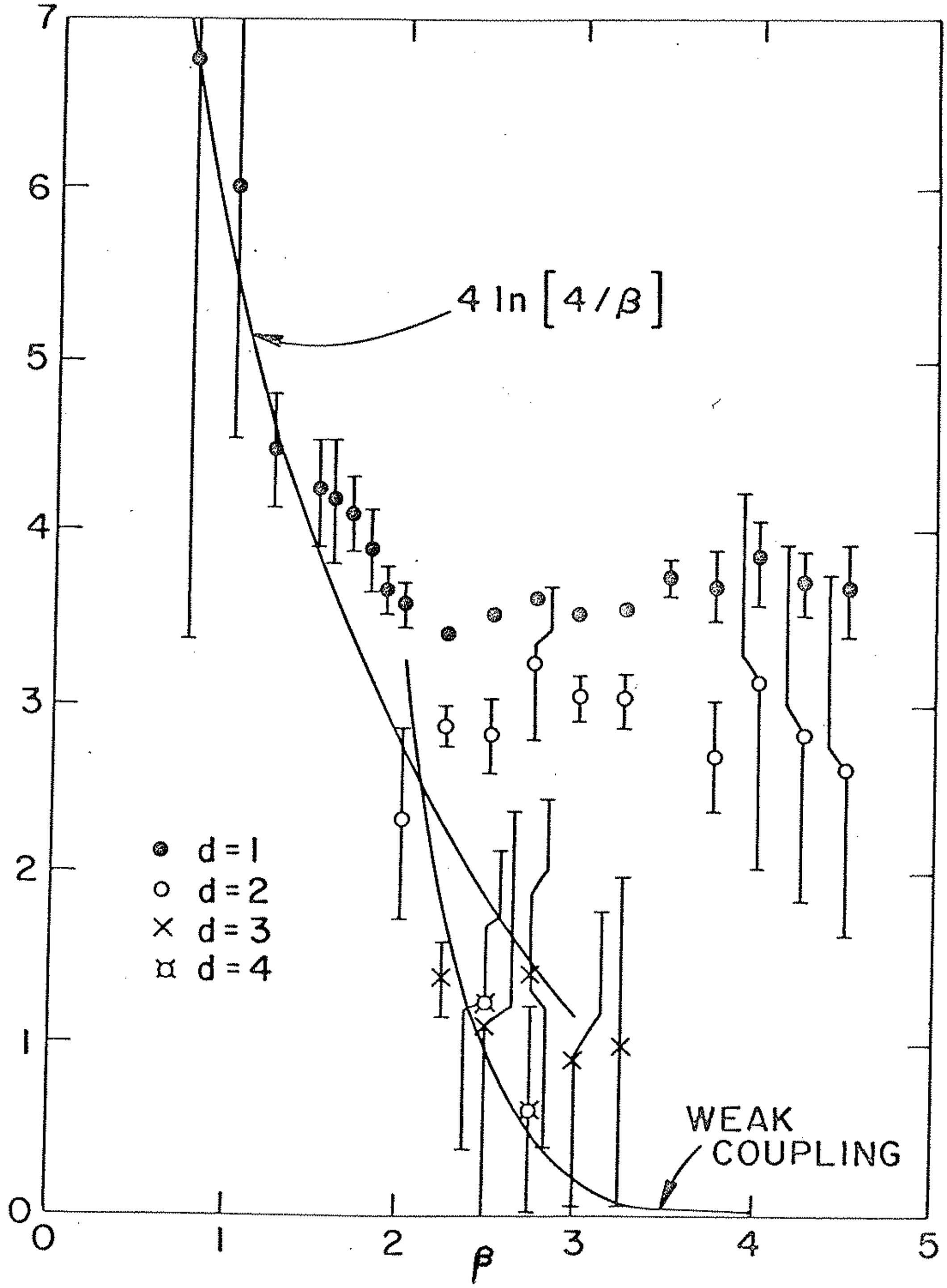


fig. 9

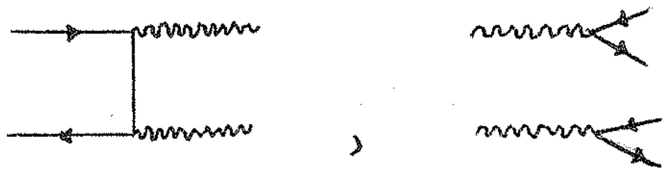
Such a fit to the above curves, allowed Bhanot and Rebbi to conclude:

$$\frac{M}{\sqrt{\sigma}} = 3 \pm 1$$

V. Glueball Phenomenology/Experimental Evidence

a. Mixing

Thus far in our discussions of the glueball mass spectrum we have often alluded to the fact that there are, in all probability, no real pure gluonic bound states because of quantum mechanical mixing with ordinary quark states of identical quantum numbers. To lowest order, the graphs responsible for this mixing are:



From past experience, we naively expect the mixing to be considerable, leading to mass effects the order of a few hundred MeV or so. Roughly speaking, if the above conjecture is true, it is unlikely that glueballs will differ very much from ordinary hadrons since, in fact, they will be partly composed of the standard $q\bar{q}$ or qqq systems. Of course the argument works both ways so that one believes some hadrons to have a sizeable glue component. In particular, recent work by Novikov, Shifman, Vainshtein, and Zakharov[33] indicates that the large mass of the pseudoscalar η' can be accounted for by assuming the particle to be a mixture of quark and glue. Their calculation reveals the matrix element $\langle \eta' | G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} | 0 \rangle$ to be much larger than expected so that the authors claim it only plausible to assume the problematic η' to be mostly glueball in character.

Although it is inevitable that there will be considerable mixing of ordinary hadrons and glueballs of identical quantum numbers, the presence of gluonic bound states should be unmistakable since each meson nonet will acquire an additional isoscalar member, thereby bringing the total to three ($\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$, $s\bar{s}$, gg). In the words of Donoghue[34] -"the SU(3) nonets of the sixties will become the QCD decuplets of the eighties!" Finally, before we begin considering the possible glueball candidates one can find in the particle data tables, it is important to make one last observation. Because the lagrangian of QCD permits gluon self-couplings, it is, strictly speaking, incorrect to separate the glueball mass spectrum into distinct 2g and 3g sectors since a simple non-Abelian gauge transformation mixes them. It is merely for aesthetic reasons and ease in classification that we allow ourselves to speak of mesonic or baryonic glueballs. The aforementioned complications should make us anticipate an experimental situation that is, at the very best, inconclusive.

b. Candidates

i. O(1100 MeV), $J^{\pi C}=1^{--}$

Searching for vector mesons in the mass region 1-2 GeV via the sequential reaction $\gamma+p \longrightarrow V^0+p, V^0 \longrightarrow e^+e^-$, the 1976 DESY/Frascati collaboration[35] discovered a seven standard deviation peak at 1100 MeV with an associated resonance width of 20 MeV. As the radial excitations of the ρ and ω were expected to lie somewhat higher in mass and to have much greater widths, the establishment of a $J^{\pi C}=1^{--}$ resonance at 1100 MeV proved to be a great embarassment to the accepted quark model. In 1977, Robson[36] suggested that the new particle was, in fact, the O, the infamous first daughter of the Pomeron, which was awaiting experimental verification as the lowest lying vector glueball. Earlier work had been done on the O by Freund and Nambu[37], who proposed that the OZI violating decays of the $\varphi(s\bar{s})$ and $\psi/J(c\bar{c})$ proceeded via a mixing of these states with the O. From their assumptions that the glueball Regge trajectories are linear with universal slope $\frac{1}{3}-\frac{1}{2}$ GeV^{-1} and that the Pomeron intercept is 1 with its daughters integer spaced below, Freund and Nambu were first able to calculate the mass of the O to be 1.4–1.8 GeV. Next, hypothesizing that the only poles necessary in their OZI violation calculations were the O, ω, φ, and ψ and that the transition amplitudes, f_{OV} ($V=\omega,\varphi,\psi$), obeyed the flavor SU(4) relations,

$$f_{O\psi} = f_{O\varphi} = \frac{1}{\sqrt{2}} f_{O\omega} = f$$

it was possible for them to obtain an estimate for the ratio $\frac{\Gamma(\psi\to\rho\pi)}{\Gamma(\varphi\to\rho\pi)}$ that was independent of f. Since the width $\Gamma(\varphi\to\rho\pi)$ was reliably known at the time, Freund and Nambu computed $\Gamma(\psi\to\rho\pi)$ to be 12.7 keV.

Motivated by the DESY/Frascati discovery of the 1100 MeV resonance and some refined experimental data which established $\Gamma(\psi\to\rho\pi)\approx 0.8$ keV, Robson was convinced that the O glueball of Freund and Nambu had been found. Employing, with some modifications, the Freund-Nambu sequential pole model delineated above, Robson set $M_O=1100$ MeV, $\Gamma_O=20$ MeV (this corresponded to a Pomeron slope just greater than $\frac{1}{2}$ for $t>0$), used the SU(4) broken couplings:

$$\frac{f_{O\psi}}{M_O+M_\psi} = \frac{f_{O\varphi}}{M_O+M_\varphi} = \sqrt{2}\,\frac{f_{O\omega}}{M_O+M_\omega}$$

and made narrow resonance approximations in all phase space calculations to ascertain a new value for the ψ/J decay width into the $\rho\pi$ channel, under the assumption that the DESY/Frascati 1100 MeV resonance was, indeed, the lowest lying vector glueball. Robson obtained the value

$$\Gamma(\psi \longrightarrow \rho\pi) \approx 1.6\ \mathrm{keV}$$

which was quite close, just a factor of two above the recently established experimental value. The success of this calculation induced Robson to suggest that the 0 glueball of Freund and Nambu had finally been verified experimentally.

ii. 'the Pomeron', $J^{\pi C}=2^{++}$

As it stands, the meson 2^{++} nonet is well documented by the particle physicists. Yet, since the MIT bag model predicts $M_{0^{++},2^{++}} \approx 1260$ MeV, whilst the Hamiltonian lattice QCD work of Kogut, Sinclair and Susskind requires $\frac{M_T}{M_S} \approx 1$, we expect a $J^{\pi C}=2^{++}$ glueball to fall right at the f(1270) mass. An examination of the branching ratios for the radiative decays of the ψ/J reveals some intriguing numbers, however, since the f(1270) is strongly seen, whilst the f'(1516) is not:

$$BR(\psi \longrightarrow \gamma f) = (2.0 \pm 0.7) \times 10^{-3} \quad \text{PLUTO}$$
$$(1.1 \pm 0.3) \times 10^{-3} \quad \text{DASP}$$

$$BR(\psi \longrightarrow \gamma f') < 0.35 \times 10^{-3}$$

This experimental situation is disturbing because, under the assumption that the two gluon couplings to u,d, and s quarks are all equal (flavorblind), we anticipate a ratio $\gamma f : \gamma f' \approx 2$. A second problem associated with this nonet is that the f $\left(=\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\right)$ has a mass lower than one would expect- some 40 MeV below the A2 $\left(=\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}\right)$, which is rather odd since the isoscalar member should be slightly heavier, not lighter than its octet partner. In any case, the crucial question is- where does the tensor glueball go? The MIT bag model prediction and the above branching ratios assure us that the 2^{++} glueball lies at a mass much lower than the f', while good phase shift information below the f(1270) requires $M_{2^{++}} \gtrsim M_f$. It is possible that the glueball could lie somewhere between the f and f', provided that its coupling to the $\pi\pi$ channel were weak, but Donoghue[34] contends that if this were true the ψ/J radiative transition experiments would've seen it. As a last resort, one can hypothesize that the tensor glueball lies, quite literally, under the f(1270) peak- an assumption that would not only bring joy to the MIT bag model theorists, but would also explain the strong $\psi/J \rightarrow \gamma f$ signal and the mass shift of the f.

iii. the SLAC E, $J^{\pi C}=1^{++}$

It is a certainty that every meson spectroscopist in this country has his own divine opinion regarding the SLAC E(1420), our last and most recent glueball candidate. It is the belief of Donoghue[34] that, if the $J^{\pi C}=1^{++}$ assignment stands weathering, the ordinary $q\bar{q}$ meson interpretation, with the E ($=s\bar{s}$) contained in the same axial vector nonet as the A1 and $D^0(1285)$ $(=\frac{u\bar{u}+d\bar{d}}{\sqrt{2}})$, is most likely correct. Nevertheless, he appears somewhat perplexed by the fact that the $\psi/J \rightarrow \gamma E$ signal is quite strong, while the D is not seen at all in the radiative decays of charmonium, since universality of the gluon couplings would require the D signal to be twice as great as that of the E! Scharre[38] agrees with Donoghue, but notes that if the true assignment is $J^{\pi C}=0^{--}$, it is highly probable that the SLAC E is a glueball because the pseudoscalar nonet is quite complete. Meshkov[39] adamently denies the

SLAC E glueball status on the grounds that the resonance is not narrow enough- his calculations indicate that one needs a width ~ 3 MeV for spin 0 and spin 1 glueballs, rather than the 50 MeV width observed for the SLAC E. Finally, work by Carlson et al.[40] seems to have validated the $J^{\pi C}=1^{++}$ assignment for the SLAC E, theoretically accounted for and reduced the "surprisingly" large branching ratio of the ψ/J into the γE channel, and made it feasible to dismiss the SLAC E(1420) as a plausible glueball candidate.

c. Production Mechanisms

Donoghue[34] suggests that the best place to search for glueballs is in the radiative decay of charmonium (cf. fig. 10).

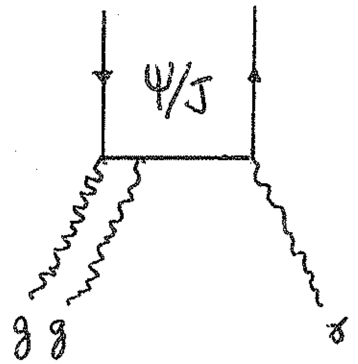
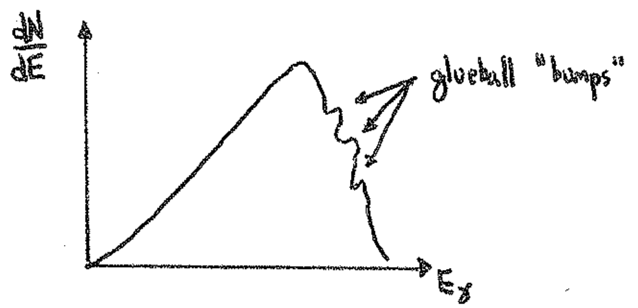


fig. 10

Since the photon energy is variable, it is conceivable that one could sweep M_{2g}^{eff} over a continuous range of masses so that if glueballs truly existed in the energy region being explored, they could be "picked out" via the E_γ tuning mechanism. Easily seen in a plot of $\frac{dN}{dE}$ vs. E_γ would be structures corresponding to these 2g resonances:

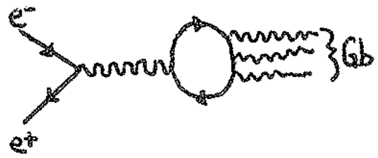


One must remember, however, that strictly speaking any single particle state seen in the decay $\psi/J \rightarrow \gamma X$, such as η, η', f(1270) or E(1420) is a partial gg resonance, which implies that it certainly has a non-negligible glueball component.

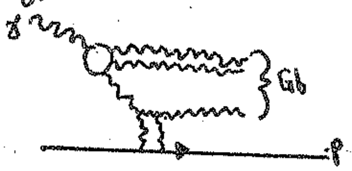
To look for the lowest lying vector glueball, we can employ any one of the traditional photoproduction mechanisms,

quark loop:

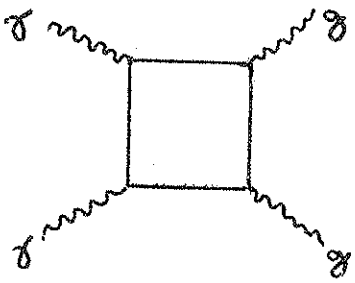
e^+e^-:



diffractive scattering:



Finally, Donoghue optimistically notes that with the increasing interest in two photon physics at PEP and PETRA, soon it will be possible to search for 2g gluonic bound states in 2γ-2g coupling processes,

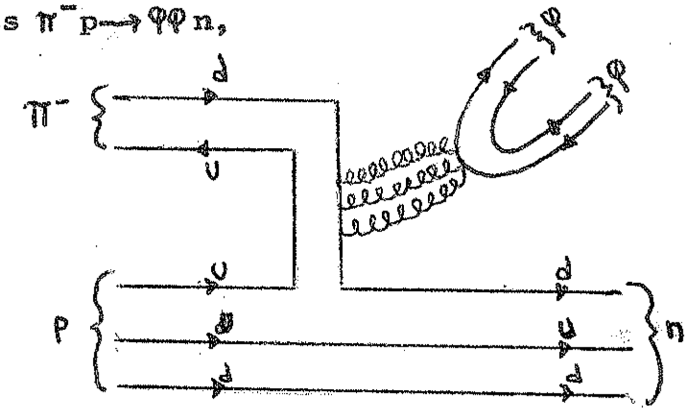


Recently, Roy and Walsh[41] proposed looking for glueballs in collinear gluon jets resulting from the decay of the heavy Υ meson. Typically in the disintegration of the Υ one expects three gluons emerging symmetrically at 120° intervals becoming jets because of the equal distribution of kinetic energies. The authors here hypothesize that if one examined the alternative process,

$\Upsilon \rightarrow 3g \rightarrow$ low sphericity final state (asymmetric collinear jets)

in which the two lower energy gluons go off in opposite directions, whilst the third, possessing the maximum kinetic energy possible, dashes off to one side and can, because it is so energetic, easily induce the creation of gluon pair from the vacuum, many glueballs could be seen since the energetic gluon could then capture one of the created pair to form a 2g bound state. Estimates made by Roy and Walsh indicate that some 80% of all such gluon jets should contain glueballs.

Lastly, Lindenbaum[42] contends that one can find glueballs in the process $\pi^- p \rightarrow \varphi\varphi n$,



where the OZI suppressed $\varphi\varphi$ system owes its origins to two gluon exchange if in a relative scalar state and to three gluon exchange if in a relative vector state. In a 1978 BNL/CCNY collaboration[43], of which Lindenbaum was part, the physicists discovered absolute violations of the OZI rule in the above process. Just recently, Lindenbaum has attempted to explain the experimental absence of suppression by postulating the intervention of actual glueball resonances, which would permit the OZI rule to be overcome. Since the multi-gluon state that gives rise to the $\varphi\varphi$ system is of variable mass, Lindenbaum suggests that, by tuning the process, one could preferentially produce the glueballs.

VI. Conclusions

In a very recent note, Donoghue, Johnson and Li[44] bring the MIT bag model treatment of the glueball mass spectrum up to date and in the process of doing so they make a number of interesting points. Firstly, motivated by the work done by Yang[45] in the fifties on possible 2γ bound states permitted by gauge and Lorentz invariance, Donoghue, Johnson and Li contend that there cannot exist a low lying $J^{\pi C}=1^{-+}$ glueball in the 2g sector. Secondly, they essentially deny Robson's objection to the bag description of 3g glueballs as unfounded. It is their belief that the use of an $l=0$, $J^{\pi}=1^{\mp}$ lowest mode, rather than the bag's $l=1$, $J^{\pi}=1^{\pm}$, is incorrect because for massless fields, the zero orbital angular momentum mode is the Coulomb mode, which doesn't exist save in the presence of external sources. Thirdly, Donoghue, Johnson and Li suggest that, despite the work of Carlson et al.[40], the SLAC 1420 resonance is, in fact, a pseudoscalar glueball (they christen it the "G") having absolutely no relation whatsoever to the problematic E(1420) seen for many years in hadronic reactions. Work by Chanowitz[46] supports this interpretation. Lastly, the bag physicists performed some calculations which give further evidence that the tensor glueball lies beneath the f(1270).

As is clear from the space devoted here to the MIT bag model, it is the belief of this author that the bag provides a reasonable analysis of the glueball mass spectrum. Granted, the original calculations of Jaffe and Johnson[15] are somewhat primitive (since the theorists neglect spin-spin splittings their estimates are on par with the ordinary hadron mass computations of Chodos, Jaffe, Johnson and Thorn). Yet, one expects that when gluon self-couplings are incorporated into the bag treatment of glueballs, a much more refined picture of the spectrum will be possible (recall fig. 1). Some work has been done in this direction by Thorn[47]. Indeed, it was via his efforts that the lowest lying 2g tensor glueball was pushed up to 1260 MeV.

With regard to the other investigatory methods employed in the study of gluonic bound states, perhaps the use of low energy theorems, within the framework of QCD, is the most promising since it has already permitted Soviet physicists to obtain a reasonable mass estimate for the lowest lying scalar gluonium, as well as account for the large mass of the pseudoscalar η'-meson. In any case, it is a certainty that strong coupling expansions for the glueball mass in the Hamiltonian formulation of lattice QCD are a thing of the past. Unless someone creates an appropriate computer algorithm, there is little hope that the y^5, y^6 and y^7 terms in the expansion of Kogut, Sinclair and Susskind[21] can be calculated within the next decade!. The situation is none too different in the Euclidean formulation of the lattice theory, save for the fact that things are a bit more subtle since one must visualize the "box" graphs of the cluster expansion in 4-space if one wants a glueball mass estimate in the real world limit. Lastly, as is apparent from the work of D'Hoker[28], Berg[29] and Bhanot and Rebbi[30], the utilization of Monte Carlo techniques to numerically determine the mass gap is, to put the matter quite bluntly, a very tricky business with a lot of fiddling going on.

The experimental situation is bleak. Nevertheless, the optimistically inclined contend that there do exist, in fact, two viable glueball candidates. It is the belief of this author that much effort should be devoted to establishing the spin and parity assignment of the SLAC γ G. In addition, a search should be initiated for the 2^{++} glueball hiding beneath the f(1270) peak. At the present moment, however, both this tensor glueball and its pseudoscalar relative remain mere conjecture. All in all, the crucial question has become- <u>where are the glueballs</u>?

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Lastly, much thanks is given by this author to Professor David Gross for his tremendous patience and very generous assistance in this project, as well as the many, many helpful insights that he provided.