Numerical Evidence for the Observation of a Scalar Glueball

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We compute from lattice QCD in the valence (quenched) approximation the partial decay widths of the lightest scalar glueball to pairs of pseudoscalar quark-antiquark states. These predictions and values obtained earlier for the scalar glueball's mass are in good agreement with the observed properties of $f_J(1710)$ and inconsistent with all other observed meson resonances.

It is generally believed that QCD predicts the existence of glueballs, resonances composed mainly of chromoelectric field without a valence quark-antiquark pair, occurring either as physical particles by themselves or in linear combination with states which do include a valence quark and antiquark. Whether such states have been identified so far in experiment remains ambiguous. A crucial problem is that the properties of glueballs are not expected to be drastically different from the properties of flavor singlet bosons including valence quarks and antiquarks. Thus the identification in experiment of states with large glueball contributions is difficult if not impossible in the absence of a reliable evaluation of the properties predicted for glueballs by QCD. We believe the lattice formulation of QCD provides the most reliable method now available for determining QCD's predictions for the masses and decay couplings of hadrons.

Some time ago we reported [1] a value of 1740(71) MeV for the valence (quenched) approximation to the infinite volume continuum limit of lattice QCD predictions for the mass of the lightest scalar glueball. This result was obtained using ensembles of 25 000 to 30 000 gauge configurations on each of several different lattices. An earlier independent valence approximation calculation [2], when extrapolated to the continuum limit [3] following Ref. [1], yields 1625(94) MeV for the lightest scalar glueball mass. This calculation used several different lattices with ensembles of between 1000 and 3000 configurations each. If the two mass evaluations are combined, taking into account the correlations between their statistical uncertainties arising from a common procedure for converting lattice quantities into physical units, the result is 1707(64) MeV for the scalar glueball mass. Both the mass prediction with larger statistical weight and the combined mass prediction are in good agreement with the mass of $f_J(1710)$ and are strongly inconsistent with all but $f_0(1500)$ [4] among the established flavor singlet resonances which could be scalars. For $f_0(1500)$ the disagreement is still by more than 3 standard deviations.

The valence approximation, used in the mass calculation of Refs. [1,2], may be viewed as replacing the momentum and frequency dependent color dielectric constant arising from quark-antiquark vacuum polarization with its zero-momentum, zero-frequency limit [5]. This approximation is expected to be fairly reliable for long-distance properties of hadrons. For example, the infinite volume continuum limits of the valence approximation to the masses of eight low-lying hadrons composed of quarks and antiquarks differ from experiment by amounts ranging up to 6% [6]. A 6% error in the glueball mass would be 100 MeV and, according to an adaptation of an argument giving a negative sign for the valence approximation error in f_π [6], the sign of this error is also expected to be negative. Alternatively, the contribution to this error arising from ignoring glueball mixing with states carrying a valence quark-antiquark pair we believe will be negative with magnitude less than 60 MeV. This estimate follows, in part, from an examination [3] of the splittings between related isosinglet and isovector meson pairs with the same one unit of orbital angular momentum as scalar quark-antiquark states. Thus the scalar glueball in full QCD should lie within 100 MeV above the valence approximation mass, and correcting the error in the valence approximation should not drastically change the comparison with experiment.

The most likely interpretation of $f_0(1500)$, we believe, is not as a glueball [7] but as a state composed largely of an $s\bar{s}$ quark-antiquark pair. The $s\bar{u}$ scalar and tensor are nearly degenerate at about 1430 MeV. Thus the $s\bar{s}$ scalar and tensor should lie close to each other somewhere above 1430 MeV. Since the $s\bar{s}$ tensor has been identified at 1525 MeV, an $s\bar{s}$ scalar at 1500 MeV would be quite natural.

The crucial question not answered by the mass results, however, is whether the decay width of the lightest scalar glueball is small enough for this particle actually to be identified in experiment. In addition, it is sometimes argued that since glueballs are flavor singlets they should have the same couplings to $2\pi_0$, to $2K_L$, and to 2η. This expectation is violated by $f_J(1710)$ decay couplings.

In the present article we report the first lattice QCD calculation of the valence (quenched) approximation to the partial decay widths of the lightest scalar glueball to pairs of pseudoscalar quark-antiquark states. The calculation is done with 10 500 gauge configurations on a single lattice, $16^3 \times 24$, at $\beta = 5.70$ corresponding to inverse lattice spacing $a^{-1} = 1.35$ GeV. We believe this lattice has spacing sufficiently small and volume sufficiently large to give partial widths within 30% of their infinite volume

continuum limits. The predicted decay couplings, combined with the mass prediction of 1740(71) MeV, give a total two-pseudoscalar decay width of 108(29) MeV for the scalar glueball. With any reasonable guess concerning the scalar glueball's branching fraction to multibody decay modes, the resulting total decay width is well below 200 MeV and therefore small enough for the scalar glueball to be identified in experiment. In fact, the predicted total two-pseudoscalar decay width, and individual couplings to $2\pi_0$, to $2K_L$, and to 2η are all in good agreement with properties of $f_J(1710)$ and inconsistent with all other established flavor singlet scalar resonances. A comparison of our results with data for $f_J(1710)$ [8] is shown in Fig. 1.

Glueballs found in the valence approximation, according to one simple interpretation, contain no admixture of configurations with valence quarks or antiquarks. Thus we consider the agreement between the mass and decay couplings found in the valence approximation and the observed mass and decay couplings of $f_J(1710)$ to be strong evidence that this state is largely a scalar glueball with at most some relatively smaller amplitude for configurations including valence quark-antiquark pairs.

The calculations presented here were carried out on the GF11 parallel computer [9] at IBM Research Center and took approximately 2 yr to complete at a sustained computation rate of between 6 and 7 Gflops. A preliminary version of this work is discussed in Ref. [10].

In the remainder of this paper we describe our method for determining scalar glueball decay couplings and then present our numerical results.

To evaluate glueball decay couplings we work with a Euclidean lattice gauge theory, on a lattice $L^3 \times T$, with the plaquette action for the gauge field, and the Wilson action for quarks. It is convenient initially to assume exact flavor SU(3) symmetry for the quark mass matrix. With each gauge configuration fixed to a lattice Coulomb gauge, we construct a collection of smeared fields. We describe smearing only for the particular choice of parameters actually used in the decay evaluation. Let $U_i(x)$ for a space direction $i = 1, 2, 3$, be a smeared link field [1] given by the average of the

FIG. 1. Decay couplings.

nine links in direction i from the sites of the (3 site) $\times$ (3 site) square oriented in the two positive space directions orthogonal to i starting at site x. Let $V_{ij}(x)$ be the trace of the product around the outside of a (3 link) $\times$ (3 link) square. Define the zero-momentum scalar glueball operator $g(t)$ to be the sum of the $V_{ij}(x)$ for all i, j, and x with time component t. Let the quark and antiquark fields $\overline{\Psi}(x)$ and $\Psi(x)$ be Wilson quark and antiquark fields smeared [6] by convoluting the local Wilson fields with a space direction Gaussian, invariant under lattice rotations and with mean-square radius 6.0. The smeared pseudoscalar field $\pi_i(x)$ with flavor index i is $\overline{\Psi}(x)\gamma^5\Lambda_i\Psi(x)$, where Λ_i is a Gell-Mann flavor matrix. Let $\tilde{\pi}_i(\vec{k}, t)$ be the Fourier transform of $\pi_i(x)$ on the time t lattice hyperplane.

Define E_1^π and E_2^π to be the energy of a single pseudoscalar at rest or with momentum magnitude $|\vec{k}| = 2\pi/L$, respectively. The field strength renormalization constant η_1^π is defined by the requirement that for large t the vacuum expectation value $\langle \tilde{\pi}_i^\dagger(0, t)\tilde{\pi}_i(0, 0) \rangle$ approaches $(\eta_1^\pi)^2 L^3 \exp[-E_1 t]$. Define η_2^π similarly from a pseudoscalar field with momentum magnitude $|\vec{k}| = 2\pi/L$. In the valence approximation, the glueball is stable so that its mass E^g and field strength renormalization constant η^g can be defined by the requirement that, for large t, $\langle g(t)g(0) \rangle$ approaches $(\eta^g)^2 L^3 \exp(-E^g t)$.

From pseudoscalar fields at position 0 and times t_i, define the two-pseudoscalar, flavor singlet field $\Pi(t_1, t_2)$ to be $(16)^{-1/2} \sum_i \pi_i(0, t_1)\pi_i(0, t_2)$, where the sum over i runs from 1 to 8. Let the zero-momentum, two-pseudoscalar flavor singlet field $\tilde{\Pi}_1(t_1, t_2)$ be $(16)^{-1/2} \sum_i \tilde{\pi}_i(0, t_1)\tilde{\pi}_i(0, t_2)$. Define the two-pseudoscalar field $\tilde{\Pi}_2(t_1, t_2)$ to be $(24)^{-1/2} \sum_{i\vec{k}} \tilde{\pi}_i(\vec{k}, t_1)\tilde{\pi}_i \times (-\vec{k}, t_2)$, where the sum for $\vec{k}$ is over the three positive orientations with $|\vec{k}| = 2\pi/L$.

Let $|1\rangle$ and $|2\rangle$ be, respectively, the lowest and second lowest energy flavor singlet, rotationally invariant two-pseudoscalar states. Both states are normalized to 1. Let $E_i^{\pi\pi}$ be the energy of $|i\rangle$. Define the amplitudes $\eta_{ij}^{\pi\pi}(t)$ to be $L^{-3}\langle i|\tilde{\Pi}_j(t, 0)|\Omega\rangle$. For large t, $\eta_{ij}^{\pi\pi}(t)$ has the asymptotic form $\eta_{ij}^{\pi\pi}\exp(-E_j^\pi t)$. The diagonal coefficients $\eta_{11}^{\pi\pi}$ and $\eta_{22}^{\pi\pi}$ are expected to be larger than the off-diagonal $\eta_{21}^{\pi\pi}$ and $\eta_{12}^{\pi\pi}$, respectively. As a consequence of the interaction between pairs of pseudoscalars, however, the off-diagonal coefficients will not be zero.

Connected three-point functions from which coupling constants can be extracted are now given by $T_i(t_g, t_\pi)$ defined as $\langle g(t_g)\tilde{\Pi}_i(t_\pi, 0)\rangle - \langle g(t_g)\rangle\langle\tilde{\Pi}_i(t_\pi, 0)\rangle$. If the quark mass, and thus the pseudoscalar mass, is chosen so that $E_1^{\pi\pi}$ is equal to E^g, the lightest intermediate state which can appear between the glueball and pseudoscalars in a transfer matrix expression for $T_1(t_g, t_\pi)$ is $|1\rangle$. Thus for large enough t_g with t_π fixed, $T_1(t_g, t_\pi)$ will be proportional to the coupling constant of a glueball to two pseudoscalars at rest. If the quark mass is chosen so that $E_2^{\pi\pi}$ is equal to E^g, however, the lightest
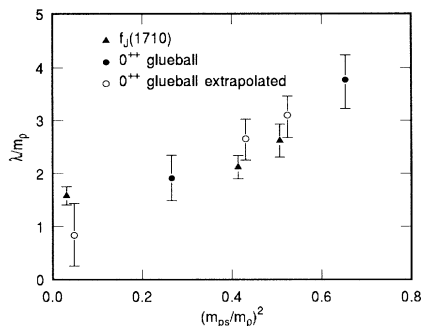
intermediate state which can appear between the glueball and pseudoscalars in a transfer matrix expression for $T_2(t_g, t_\pi)$ we still expect to be $|1\rangle$, not $|2\rangle$, since $\eta_{12}^{\pi\pi}(t)$ is expected not to be zero. To obtain from $T_2(t_g, t_\pi)$ the coupling of a glueball to two pseudoscalars with momenta of magnitude $2\pi L^{-1}$, the contribution to $T_2(t_g, t_\pi)$ arising from the $|1\rangle$ intermediate state must be removed.

From the three-point functions we therefore define the amplitudes

$$S_i(t_g, t_\pi) = T_i(t_g, t_\pi) - \frac{\eta_{ji}^{\pi\pi}(t_\pi)}{\eta_{jj}^{\pi\pi}(t_\pi)} T_j(t_g, t_\pi), \quad (1)$$

for (i, j) of either (1,2) or (2,1). In $S_2(t_g, t_\pi)$ the contribution of the undesirable $|1\rangle$ intermediate state has been canceled. In $S_1(t_g, t_\pi)$ a contribution from the intermediate state $|2\rangle$ has been canceled. Although the subtraction in $S_1(t_g, t_\pi)$ is irrelevant for large enough t_g, we expect that as a result of this subtraction $S_1(t_g, t_\pi)$ will approach its large t_g behavior more rapidly than does $T_1(t_g, t_\pi)$.

An additional intermediate state which can also appear in a transfer matrix expression for either $T_i(t_g, t_\pi)$ is the isosinglet scalar bound state of a quark and an antiquark. For the parameter values used in the present calculation we have found that this state has a mass in lattice units above 1.25, while the scalar glueball mass is 0.972(44). Thus for large enough t_g the scalar quark-antiquark state will make only its appropriate virtual contribution and does not require an additional correction.

At large t_g and t_π, the three-point functions become

$$S_i(t_g, t_\pi) \rightarrow \frac{c_i \sqrt{3} \lambda_i \eta^g \eta_{ii}^{\pi\pi}(1 - r)L^3}{\sqrt{8E^g(E_i^\pi)^2}} s_i(t_g, t_\pi), \quad (2)$$

where $c_1 = 1/\sqrt{2}$, $c_2 = \sqrt{3}$, r is $\eta_{12}^{\pi\pi}\eta_{21}^{\pi\pi}/\eta_{11}^{\pi\pi}\eta_{22}^{\pi\pi}$, and λ_1 and λ_2 are the glueball coupling constants to a pair of pseudoscalars at rest or with momenta of magnitude $2\pi L^{-1}$, respectively. The factors $\eta_{ij}^{\pi\pi}$ are given by the large t behavior of $\eta_{ij}^{\pi\pi}(t)$ as discussed earlier. For $T \gg t_g \geq t_\pi$, the factors $s_i(t_g, t_\pi)$ are

$$s_i(t_g, t_\pi) = \sum_t \exp[-E^g|t - t_g| - E_i^\pi|t|$$
$$- E_i^\pi|t - t_\pi| - \delta_i(t, t_\pi)|t - t_\pi|], \quad (3)$$

where, for $t \geq t_\pi$, $\delta_i(t, t_\pi)$ is the binding energy $E_i^{\pi\pi} - 2E_i^\pi$ and otherwise $\delta_i(t, t_\pi)$ is 0.

The coupling constants in Eq. (2) have been identified by comparing $S_i(t_g, t_\pi)$ with the three-point functions arising from a simple phenomenological interaction Lagrangian. This procedure is correct to leading order in the coupling constants. A similar relation used to find coupling constants among hadrons containing quarks has recently yielded several predictions in good agreement with experiment [11]. The λ_i are normalized so that in the continuum limit they become, up to a factor of $-i$, Lorentz-invariant decay amplitudes with the standard nor-

malization convention used in the section on kinematics of the Review of Particle Properties.

To obtain values of λ_i from Eq. (2) we need the amplitudes $\eta_{ij}^{\pi\pi}(t)$. These we determine from propagators for two-pseudoscalar states. Define two-pseudoscalar propagators $C_i(t_1, t_2)$ to be $\langle \Pi(t_1 + 2t_2, t_1 + t_2)\tilde{\Pi}_i(t_2, 0)\rangle$. For moderately large values of t_1, these amplitudes approach

$$C_i(t_1, t_2) = C_{1i}\exp(-E_1^{\pi\pi}t_1) + C_{2i}\exp(-E_2^{\pi\pi}t_1), \quad (4)$$

$$C_{ij} = \eta_{i1}^{\pi\pi}(t_2)\eta_{ij}^{\pi\pi}(t_2) + \sqrt{6}\eta_{i2}^{\pi\pi}(t_2)\eta_{ij}^{\pi\pi}(t_2). \quad (5)$$

From these expressions the required $\eta_{ij}^{\pi\pi}(t)$ can be extracted.

The $\eta_{ij}^{\pi\pi}(t)$ in Eq. (2) serve, among other purposes, to correct for the interaction between the two pseudoscalars produced by a glueball decay. In the valence approximation this interaction does not include the production and annihilation of virtual quark-antiquark pairs. Correspondingly, in the numerical evaluation of $C_i(t_1, t_2)$ from quark propagators, we include only terms in which all initial quarks and antiquarks propagate through to some final quark or antiquark. Terms in the two-pseudoscalar propagator in which initial quarks propagate to initial antiquarks, if inserted into a glueball decay diagram, lead to closed quark loops and thus to processes missing from glueball decay in the valence approximation. For very large t_1 and T, the $C_i(t_1, t_2)$ are given by a sum of two terms each of which is a slightly more complicated version of one of the exponentials in Eq. (4). This complication occurs, for example, because in the valence approximation the exchange of a ρ between the pseudoscalars produced in a glueball decay is not iterated in the same way as in full QCD. Each term in Eq. (4) holds without modification if $|E_i^{\pi\pi} - 2E_i^\pi|^2 t_1^2/2 \ll 1$. The intervals of t_1 we use to determine the $\eta_{ij}^{\pi\pi}$ fall well within this limitation. In any case, as we will discuss below, the measured values of $\eta_{ij}^{\pi\pi}$ turn out to be close to their values for noninteracting pseudoscalars. As a consequence, the corrections due to interactions between the decay pseudoscalars which the $\eta_{ij}^{\pi\pi}$ contribute to the predicted values of λ_i are comparatively small.

We now turn to our numerical results. At $\beta = 5.7$ on a $16^3 \times 24$ lattice, with an ensemble of 10 500 independent configurations, we determined glueball and single pseudoscalar energies and renormalization constants following Refs. [1] and [6], respectively. For E^g, as mentioned above, we found 0.972 ± 0.044. On a lattice of size $16^3 \times 40$ we then evaluated the two-pseudoscalar propagator $C_i(t_1, t_2)$ at $\kappa = 0.1650$ using 100 independent configurations, and at $\kappa = 0.1675$ using 875 independent configurations. Fitting the t_1 dependence of $C_i(t_1, t_2)$ by Eqs. (4) and (5), we determined $E_i^{\pi\pi}$ and $\eta_{ij}^{\pi\pi}(t_2)$ for a range of different t_2. At $\kappa = 0.1650$ we obtained results for $0 \leq t_2 \leq 4$, and at 0.1675 we found results for $0 \leq t_2 \leq 5$. The values of $E_i^{\pi\pi}$ were statistically consistent with being independent of t_2 in all cases. The $\eta_{ij}^{\pi\pi}(t_2)$

were consistent with the asymptotic form $\eta_{ij}^{\pi\pi}\exp(-E_j^\pi t_2)$ in all cases for $t_2 \geq 2$. At $\kappa = 0.1650$ for $E_1^{\pi\pi}$ we obtained $0.908(5)$, giving glueball decay to $|1\rangle$ nearly on mass shell. At $\kappa = 0.1675$ for $E_2^{\pi\pi}$ we found $0.893^{+0.044}_{-0.004}$, giving glueball decay to $|2\rangle$ nearly on mass shell. For the normalized ratios $\hat{\eta}_{ij}^{\pi\pi}$ defined as $\eta_{ij}^{\pi\pi}/(\eta_j^\pi)^2$, at $\kappa = 0.1650$ we obtained for ij of 11, 12, 21, and 22, the values $0.988(30)$, $0.091(8)$, $-0.087(8)$, and $1.065(13)$, respectively. At $\kappa = 0.1675$ we found $1.050(21)$, $0.107(6)$, $-0.112(8)$, $1.053(53)$. For noninteracting pseudoscalars $\hat{\eta}_{ij}^{\pi\pi}$ is 1 for $i = j$ and 0 otherwise. Our data are close to these values. The final value of λ_1 is changed by less than 1 standard deviation and the final λ_2 is changed by less than 2 standard deviations if we ignore the determination of $\hat{\eta}_{ij}^{\pi\pi}$ and simply use the noninteracting values.

From our 10 500 configuration ensemble on a $16^3 \times 24$ lattice, we evaluated S_1 and S_2 for glueball decay on mass shell at κ of 0.1650 and 0.1675, respectively. We obtained statistically significant results for $0 \leq t_g - t_\pi \leq 2$ with $0 \leq t_\pi \leq 8$. At each point within this range we then determined effective λ_i using Eq. (2). We found λ_1 and λ_2 statistically consistent with being constant for $t_\pi \geq 3$ and $t_\pi \geq 2$, respectively, and all values of $t_g - t_\pi$. Figure 2(a), for example, shows effective λ_2 in units of the ρ mass as a function of t_π for $t_g - t_\pi = 2$, in comparison to a fit with $2 \leq t_\pi \leq 6$, $t_g - t_\pi = 2$. Figure 2(b) shows fitted values of λ_2 on the interval $2 \leq t_\pi \leq 6$ for fixed $t_g - t_\pi$ of 0, 1, or 2. To extract final values of λ_i, we tried fits to all rectangular intervals of data including at least four values of t_π and at least two values of $t_g - t_\pi$. For each λ_i we chose the fit giving the lowest value of χ^2 per degree of freedom. The window determined in this way for λ_1 is $3 \leq t_\pi \leq 7$ with $1 \leq t_g - t_\pi \leq 2$, and for λ_2 is $2 \leq t_\pi \leq 6$ with $0 \leq t_g - t_\pi \leq 1$. The horizontal line in Fig. 2(b) shows the final value of λ_2. Over the full collection of windows

we examined, the fitted results varied from our final results by at most 1 standard deviation. We believe our best fits provide reasonable estimates of the asymptotic coefficients in Eq. (2).

So far our discussion has been restricted to QCD with u, d, and s quark masses degenerate. An expansion to first order in the quark mass matrix taken around some relatively heavy SU(3) symmetric point gives glueball decay couplings for π's, K's, and η's, which are a common linear function of each meson's average quark mass. Since meson masses squared are also nearly a linear function of average quark mass, the decay couplings are a linear function of meson masses squared. Thus from a linear fit to our predictions for decay couplings as a function of pseudoscalar mass squared at unphysical degenerate values of quark masses we can extrapolate decay couplings for physical nondegenerate values of quark masses. From this linear fit a prediction can also be made for the decay coupling of the scalar glueball to $\eta + \eta'$, if we ignore the contribution to the decay from the process in which the η quark and antiquark are connected to each other by one propagator and the η' quark and antiquark are connected to each other by a second propagator.

Figure 1 shows predicted coupling constants as a function of predicted meson mass squared along with linear extrapolations of the predicted values to the physical π, K, and η masses, in comparison to observed decay couplings [8] for decays of $f_J(1710)$ to pairs of π's, K's and η's. Masses and decay constants are shown in units of the ρ mass. Our predicted width for the scalar glueball decay to $\eta + \eta'$ is 6(3) MeV. For the ratio $\lambda_{\eta\eta'}/\lambda_{\eta\eta}$ we get 0.52(13). We predict a total width for glueball decay to pseudoscalar pairs of 108(29) MeV, in comparison to 99(15) MeV for $f_J(1710)$.
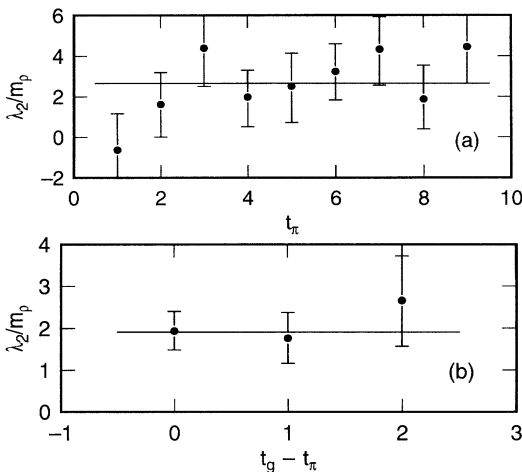

FIG. 2. (a) λ_2 for $t_g - t_\pi = 2$. (b) λ_2 fitted on $2 \leq t_\pi \leq 6$ as a function of $t_g - t_\pi$ for the fitting interval.

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