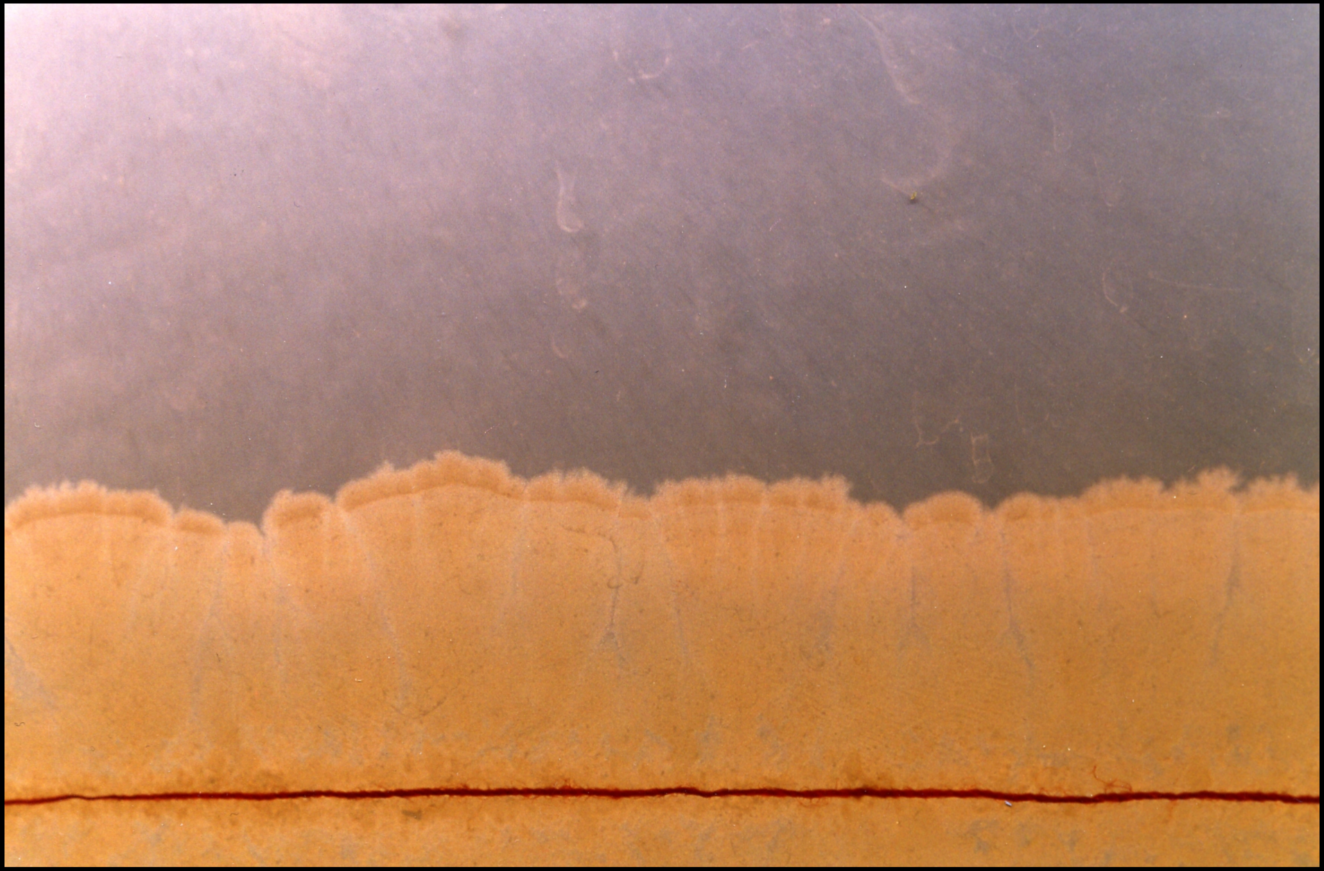
2+1 KPZ:

-George Palasantzas (Groningen)
-Kazumasa Takeuchi (Tokyo)
-Yuexia Lin (Barnard)

Universal Distributions & Correlators



Kinetic Roughening, Nonequilibrium SM, Directed Polymers in Random Media,...

2d Ising Model:

Square vs. Kagome Lattice

Onsager (1944)
Kaufmann (1949)

Nambu (Tokyo, 1949)
Husimi (Osaka, 1949)
Yamamoto (Kyoto, 1951)
Naya (Osaka, 1953)

Progress of Theoretical Physics, Vol. VI, No. 3, May～June, 1951.

Statistics of Kagomé Lattice

Itiro Syôzi
Department of Physics, Osaka University

(Received February 27, 1951)

The transition temperature of the kagomé lattice with $Z=4$ is obtained and compared with that of the square lattice.

After the work of Onsager,[1] who solved exactly the problem of Ising model for the case of plane square lattice, the same problems for the honeycomb and triangular lattice were treated by several authors.[2] Other than these three types of lattices, there is left a lattice, called in Japanese kagomé (woven bomboo pattern), which consists exclusively of equivalent lattice points and equivalent bonds. Since the number of nearest neighbors of a lattice point is as many as in the square lattice, namely four, it is interesting to verify the natural conjecture that the curie point, in general, is determined solely by the relation $\mathrm{ch}2H=\sec \pi/Z$ established by Onsager for the three types of lattices.

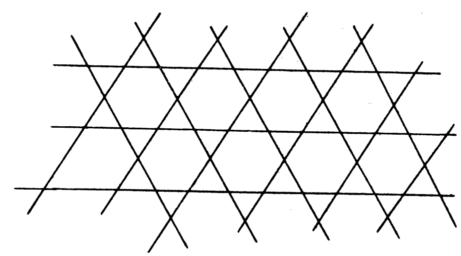
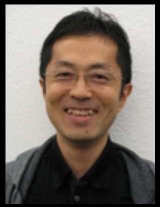


Fig. 1. Kagomé Lattice

Let us start from a variant of the honeycomb lattice, which has an extra spin on the middle point of every side as well as on every vertex (say decorated honeycomb lattice). Let its interaction parameter be L. By summing at first over the spin variables with respect to the vertices in the partition function of this lattice, we arrive at the partition function of the kagomé lattice (Star-triangle transformation), with an interaction parameter K; in fine



“Kiyomizu no butai kara tobi oriru tsumori de”

Kiyomizu-dera, Kyoto

Outline:

i) 1+1 KPZ

exps, amplitudes, LD

TW-GOE, TW-GUE, Baik-Rains F_o

ii) 2+1 KPZ Class

-Simple Height Distributions (HD)

-SLRD & EVS (local)

-Universal Limit Distribution

(2+1 analogs: TW & BR)

-Universal Spatial (Airy$_1$)

& Temporal Covariance

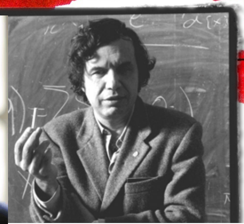
(KPZ Ageing)

KPZ PRL

≳ 2200 CITATIONS

9 PHYSICAL REVIEW LETTERS 3 MARCH 1986

25 YEAR ANNIVERSARY


Dynamic Scaling of Growing Interfaces

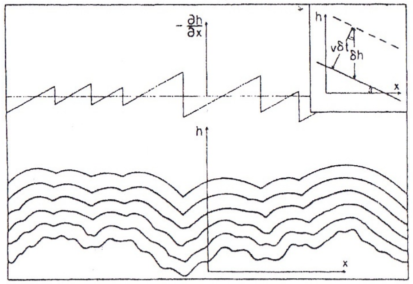
Mehran Kardar
Physics Department, Harvard University, Cambridge, Massachusetts 02138

Giorgio Parisi
Physics Department, University of Rome, I-00173 Rome, Italy

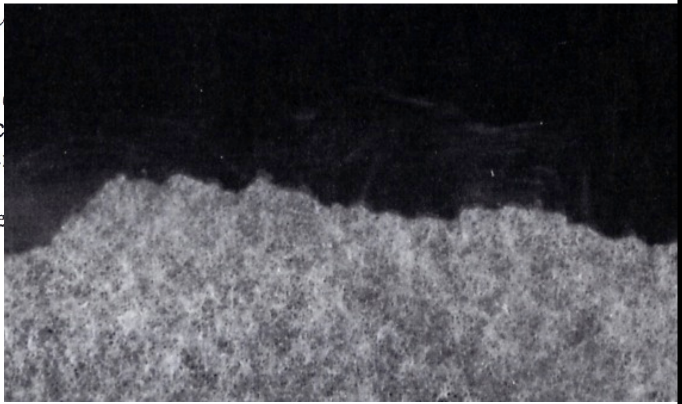
and

Yi-Cheng Zhang
Physics Department, Brookhaven National Laboratory, Upton, New Y

(Received 12 November 1985)

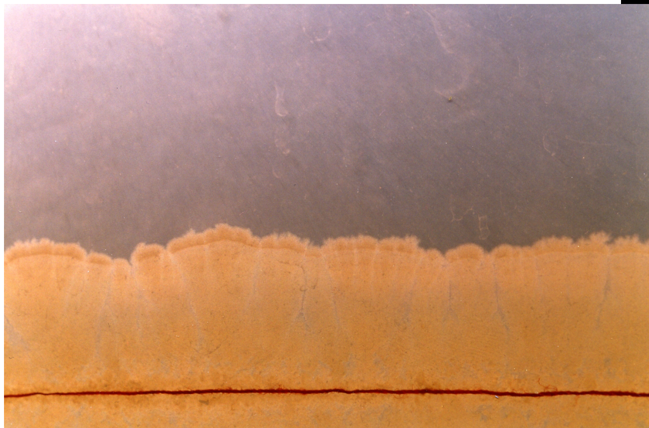


A model is proposed for the evolution of the profile of a growing interfa
growth is solved exactly, and exhibits nontrivial relaxation patterns. The stoc
ied by dynamic renormalization-group techniques and by mappings to Burge
random directed-polymer problem. The exact dynamic scaling form obtained
interface is in excellent agreement with previous numerical simulations. Pre
more dimensions.



$$\frac{\partial h}{\partial t} = \underbrace{\nu \nabla^2 h}_{\text{RELAXATION}} + \underbrace{\frac{1}{2}\lambda(\nabla h)^2}_{\text{NONLINEARITY}} + \underbrace{\eta}_{\text{STOCHASTIC NOISE}}$$

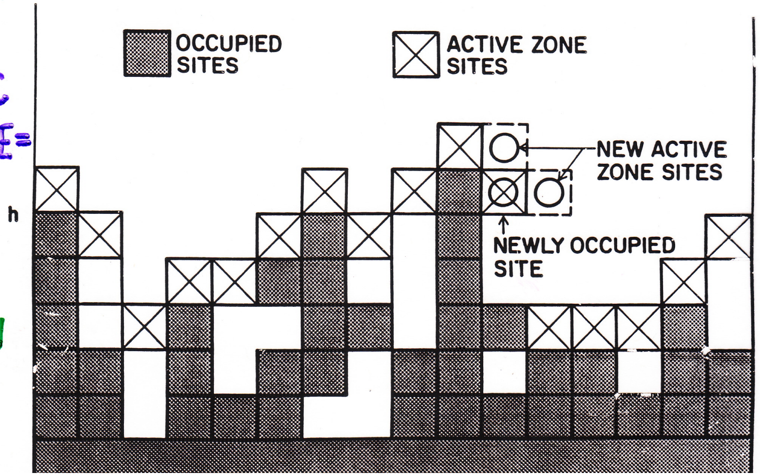
$$\langle \eta(x,t)\,\eta(x',t')\rangle = D\,\delta(x-x')\,\delta(t-t')$$



BALLISTIC DEPOSITION:

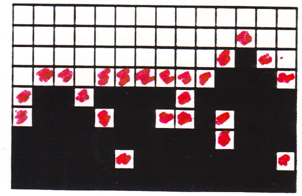
(THIN FILM GROWTH, MBE,) ~ NSF ##
TETRIS...

STOCHASTIC GROWTH RULE =
VERTICAL DROP
+
STICK UPON FIRST CONTACT

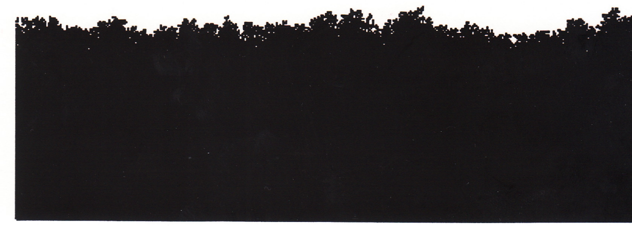


EDEN CLUSTER:

(BACTERIAL COLONY,
FOREST FIRE PROPAGATION)

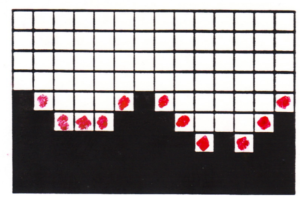


RULE?
ALL PERIMETER SITES EQUALLY LIKELY



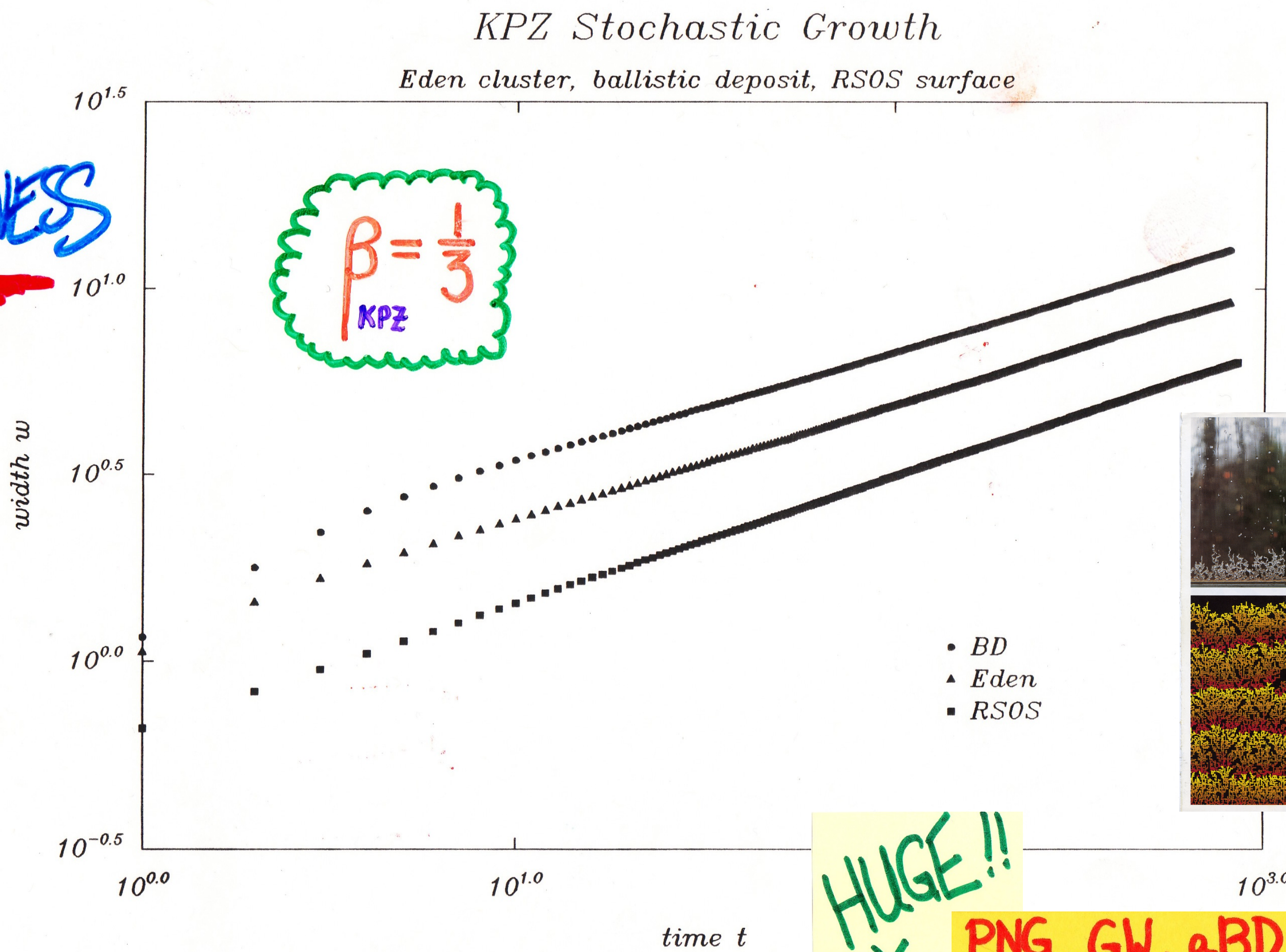
RSOS MODEL:

KIM & KOSTERLITZ
PHYS. REV. LETT. (1989)

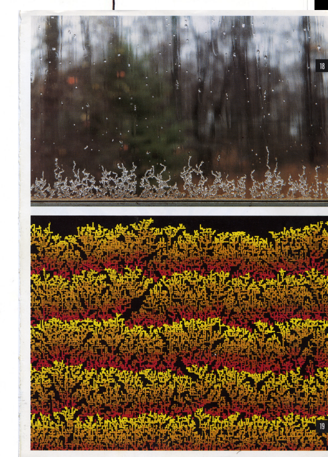


$|\Delta h| \leqslant 1$

EARLY TIME ROUGHNESS



$\beta_{KPZ} = \frac{1}{3}$



HUGE!!

PNG, GW, aBD, SS, etc...

$W \sim t^{\beta}$

→ A SINGLE UNIVERSALITY CLASS...

FLAMELESS FIRE FRONTS

Kinetic Roughening in Slow Combustion of Paper

J. Maunuksela,[1] M. Myllys,[1] O.-P. Kähkönen,[1] J. Timonen,[1] N. Provatas,[2,3] M. J. Alava,[4,5] and T. Ala-Nissila[2,6,*]

[1]*Department of Physics, University of Jyväskylä, P.O. Box 35, FIN-40351 Jyväskylä, Finland*
[2]*Helsinki Institute of Physics, University of Helsinki, P.O. Box 9, FIN-00014 Helsinki, Finland*
[3]*Department of Physics and Mechanical Engineering, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801-3080*
[4]*Laboratory of Physics, Helsinki University of Technology, P.O. Box 1000, FIN-02150 HUT, Espoo, Finland*
[5]*NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark*
[6]*Department of Physics, Brown University, Providence, Rhode Island 02912*

(Received 18 March 1997)

We present results from an experimental study on the kinetic roughening of slow combustion fronts in paper sheets. The sheets were positioned inside a combustion chamber and ignited from the top to minimize convection effects. The emerging fronts were videotaped and digitized to obtain their time-dependent heights. The data were analyzed by calculating two-point correlation functions in the saturated regime. Both the growth and roughening exponents were determined and found consistent with the Kardar-Parisi-Zhang equation, in agreement with recent theoretical work. [S0031-9007(97)03836-2]

$\chi = 0.46$

PRL 83, 2054 (1999)

KR - PENETRATING FLUX FRONTS
HIGH T_c THIN FILM SUPERCONDUCTORS
R. WIJNGAARDEN - AMSTERDAM

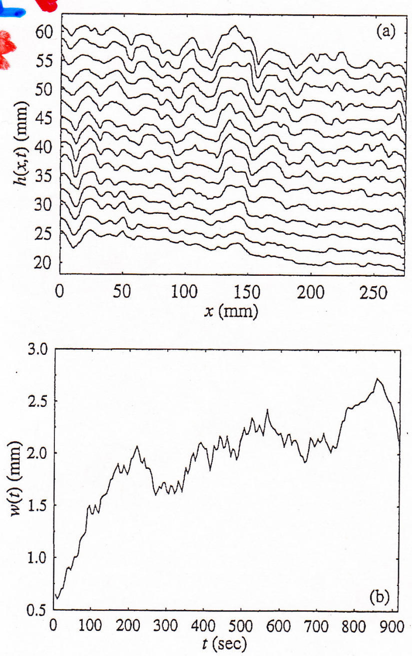


FIG. 2. (a) A series of successive digitized flame fronts taken every 5 s following the ignition of copier paper. (b) Evolution of the time-dependent surface width $w(t)$.

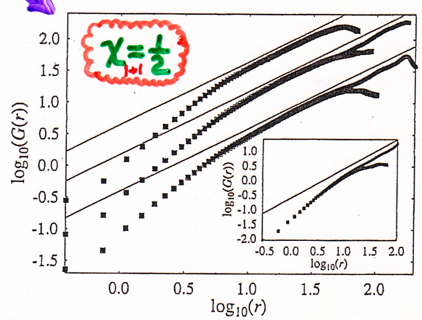


FIG. 3. The spatial correlation function $G(r)$ for three different burns of the copier paper (data have been shifted for clarity and the units are in mm). Filled circles denote the case where the average global tilt of the interface has been subtracted out. The solid lines denote $2\chi = 1$. Inset shows corresponding data for the cigarette paper.

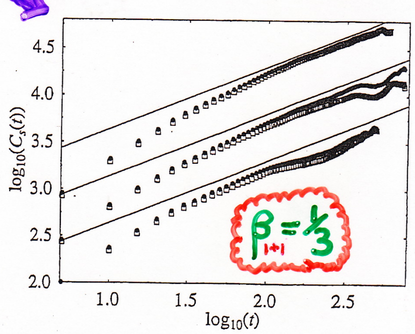
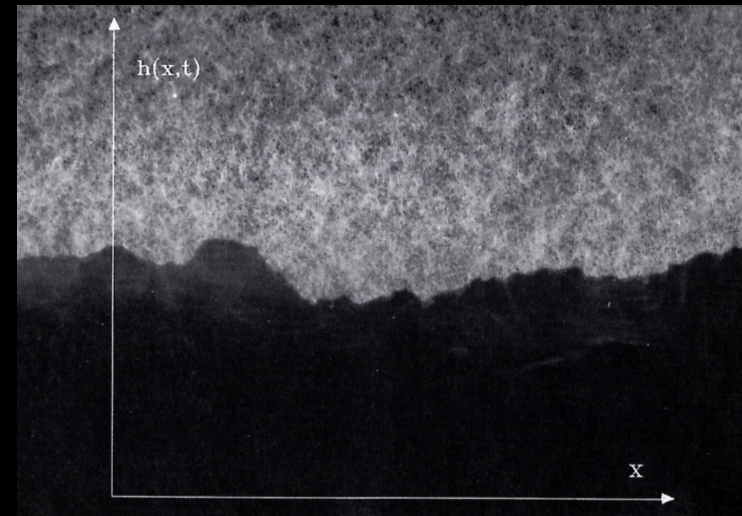


FIG. 4. Time-dependent correlation functions $C_s(t)$ for the data used in Fig. 3. The solid lines denote $2\beta = 2/3$.

also, PHYS. REV. E 64, 036101 (2001)
issue of scaling & noise
PRL 84, 1946 (2000)



JUN ZHANG, et al.
Physica A 189, 383 (1992)
"MODELING FOREST-FIRE BY PAPER BURNING EXPT"

KPZ UNIVERSALITY → RANDOM MATRICES & TRACY-WIDOM DIST.

VOLUME 84, NUMBER 21 PHYSICAL REVIEW LETTERS 22 MAY 2000

Universal Distributions for Growth Processes in 1 + 1 Dimensions and Random Matrices

Michael Prähofer* and Herbert Spohn†
Zentrum Mathematik and Physik Department, TU München, D-80290 München, Germany
(Received 14 December 1999)

We develop a scaling theory for Kardar-Parisi-Zhang growth in one dimension by a detailed study of the polynuclear growth model. In particular, we identify three universal distributions for shape fluctuations and their dependence on the macroscopic shape. These distribution functions are computed using the partition function of Gaussian random matrices in a cosine potential.

PNG MODEL:

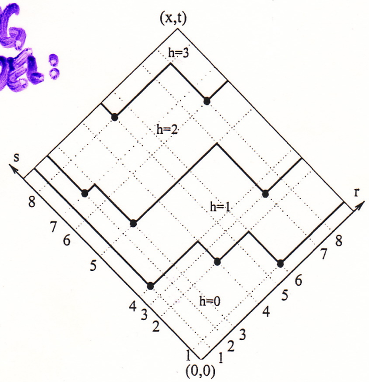


FIG. 1. The height h of a PNG droplet with nucleation events corresponding to the permutation (4, 7, 5, 2, 8, 1, 3, 6).

PNG=LIS
(Ulam Problem)
&.....
SS/DPRM/ASEP- Johannson, 2000
ODB- Gravner, Tracy & Widom, 2001
aBD- Majumdar & Nechaev, 2004

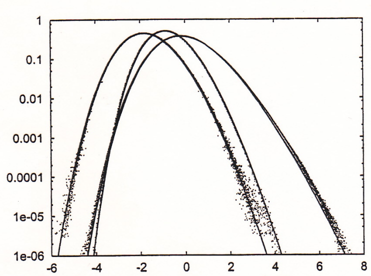


FIG. 2. From left to right: the probability densities of the universal distributions χ_2, χ_1, and χ_0 for curved, flat, and stationary self-similar growth, respectively.

Tracy-Widom Distributions, 1994:
GUE (radial geometry),
GOE (flat IC), …
RM Ensembles,

TABLE I. Mean, variance, skewness, and kurtosis for the distributions of χ_2, χ_1, and χ_0 as determined by numerically solving Painlevé II [19]. $\langle\chi^n\rangle_c$ denotes the nth cumulant.

	Curved (χ_2)	Flat (χ_1)	Stationary (χ_0)
$\langle\chi\rangle$	−1.771 09	−0.760 07	0
$\langle\chi^2\rangle_c$	0.813 20	0.638 05	1.150 39
S: $\langle\chi^3\rangle_c/\langle\chi^2\rangle_c^{3/2}$	0.2241	0.2935	0.359 41
K: $\langle\chi^4\rangle_c/\langle\chi^2\rangle_c^2$	0.093 45	0.1652	0.289 16

scaled cumulants;
skewness s & kurtosis k

Experimental determination of KPZ height-fluctuation distributions

L. Miettinen[a], M. Myllys, J. Merikoski, and J. Timonen
Department of Physics, University of Jyväskylä, P.O. Box 35 (YFL), 40014 Jyväskylä, Finland

Received 16 December 2004 / Received in final form 30 March 2005
Published online 8 August 2005 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2005

Abstract. Height-fluctuation distributions of nonequilibrium interfaces were analyzed using slow-combustion fronts propagating in sheets of paper. All distributions measured were definitely non-Gaussian. The experimental distributions for transient and stationary regimes were well fitted by the theoretical distributions proposed by Prähofer and Spohn in reference [9]. Consistent with the Galilean invariance of the system, the same distributions were found for horizontal fronts and, when determined along the normal to the slope, for fronts with a non-zero average slope.

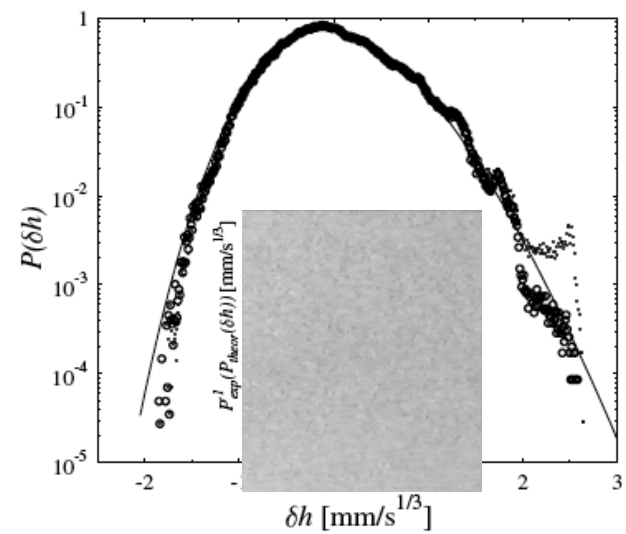
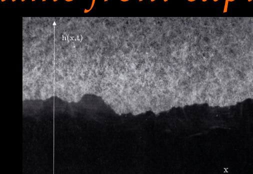


Fig. 3. Height-fluctuation distribution for horizontal fronts in the transient ($w \sim t^{1/3}$) regime, and a fit by a (scaled and shifted) theoretical distribution f_1. A theoretical inversion of the measured distribution is shown in the inset. The dots denote the measured data and the circles the data with an avalanche suppressed.

more Finnish flame front expts…
(2005)



PRL 104, 230601 (2010) PHYSICAL REVIEW LETTERS week ending 11 JUNE 2010

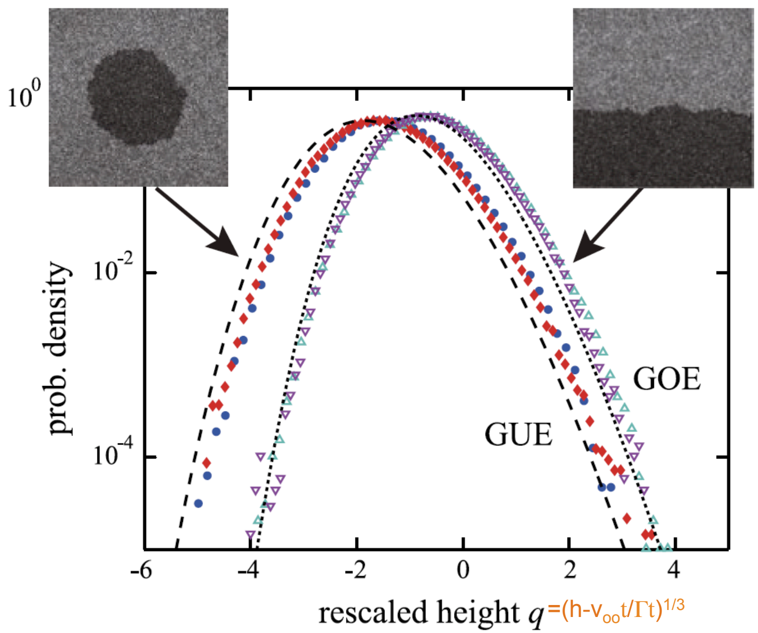
Universal Fluctuations of Growing Interfaces: Evidence in Turbulent Liquid Crystals

Kazumasa A. Takeuchi* and Masaki Sano
Department of Physics, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
(Received 28 January 2010; published 11 June 2010)

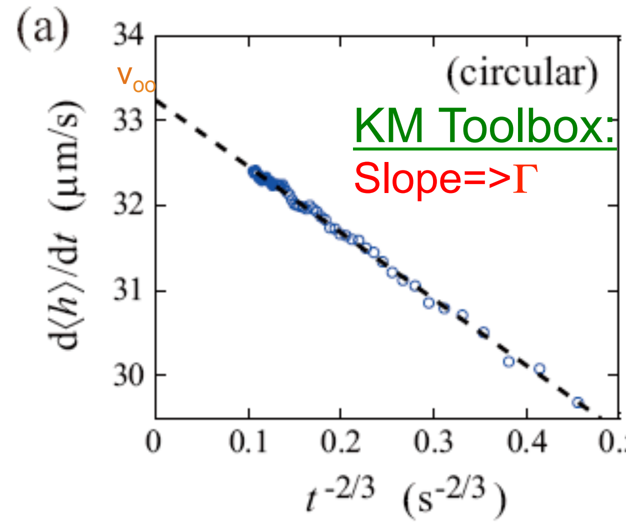
We investigate growing interfaces of topological-defect turbulence in the electroconvection of nematic liquid crystals. The interfaces exhibit self-affine roughening characterized by both spatial and temporal scaling laws of the Kardar-Parisi-Zhang theory in 1 + 1 dimensions. Moreover, we reveal that the distribution and the two-point correlation of the interface fluctuations are universal ones governed by the largest eigenvalue of random matrices. This provides quantitative experimental evidence of the universality prescribing detailed information of scale-invariant fluctuations.

Random Matrix Theory: TW Limit Distributions

Fig. 8 Histogram of the rescaled local height $q \equiv (h - v_\infty t)/(\Gamma t)^{1/3}$ for the circular (*solid symbols*) and flat (*open symbols*) interfaces. The *blue circles* and *red diamonds* display the histograms for the circular interfaces at $t = 10$ s and 30 s, respectively, while the *turquoise up-triangles* and *purple down-triangles* are for the flat interfaces at $t = 20$ s and 60 s, respectively. The *dashed* and *dotted curves* show the GUE and GOE TW distributions, respectively, defined by the random variables χ_{GUE} and χ_{GOE}. (Color figure online)



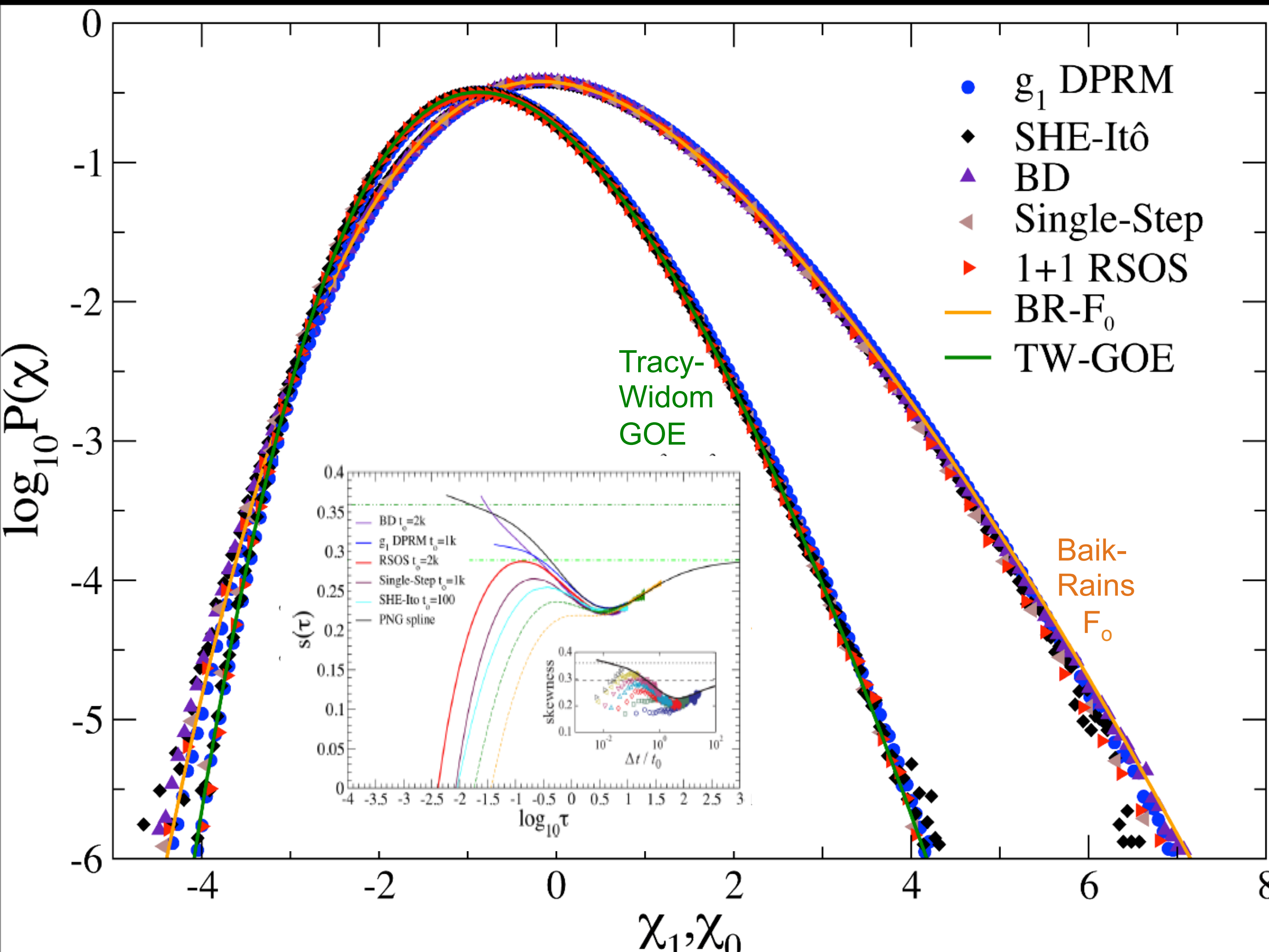
Time-Dependent Growth Velocity:



1+1 KPZ Class: Limit Distributions

(Crossover: Flat to Stationary-State Statistics)

KT-PRL**110**,210604(2013)
THH/LL-PRE**89**,010103(2014)

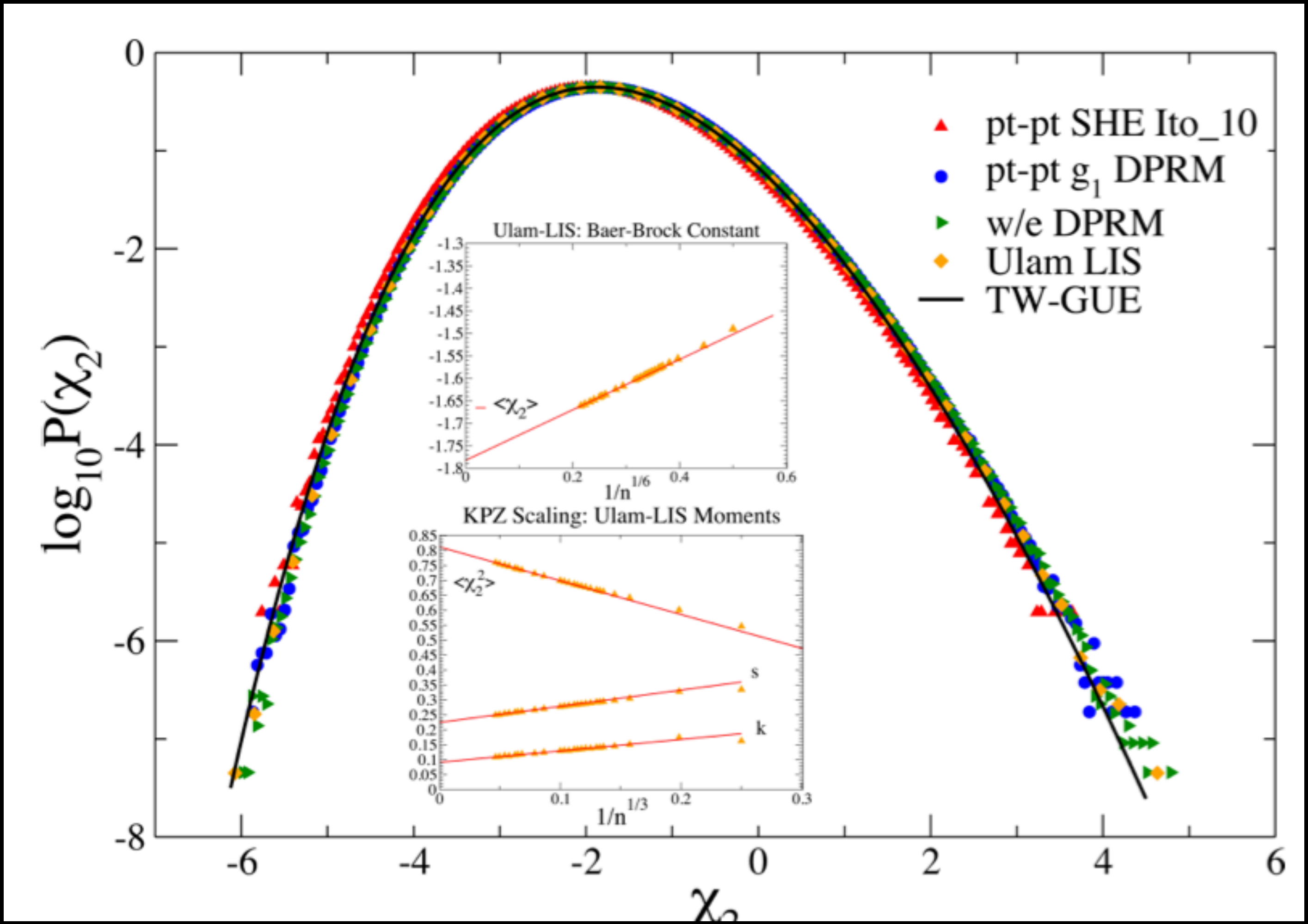


KPZ TW-GUE*
LIS Finite Time
Corrections...

KPZ Radial Class: Limit Distribution

(Interplay: TW & KPZ)

THH/LL-PRE**89**,010103(2014)



KPZ TW-GUE*

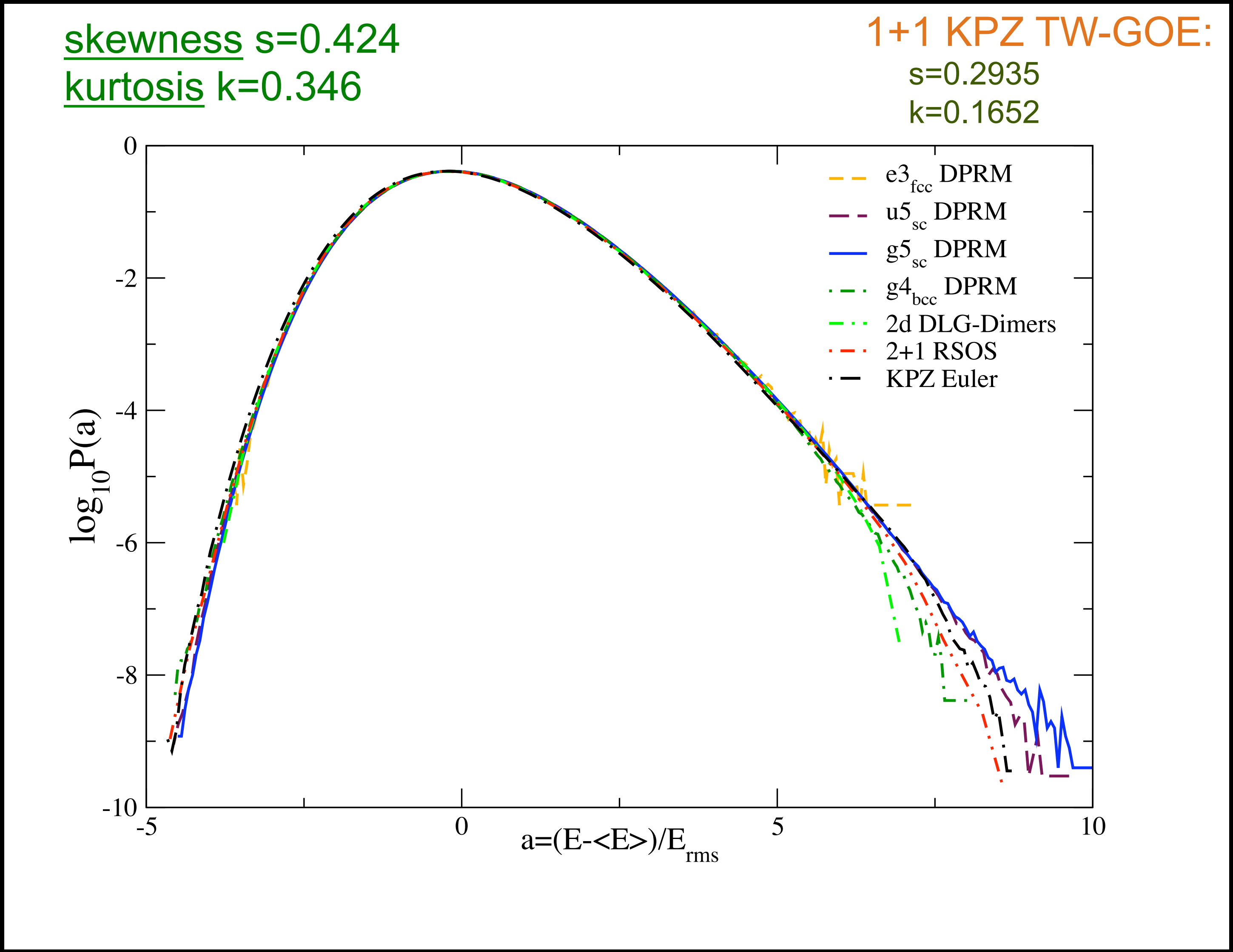
LIS Finite-Time Corrections...

2+1 KPZ

Universal Distributions…

2+1 KPZ Universality: Height PDF-

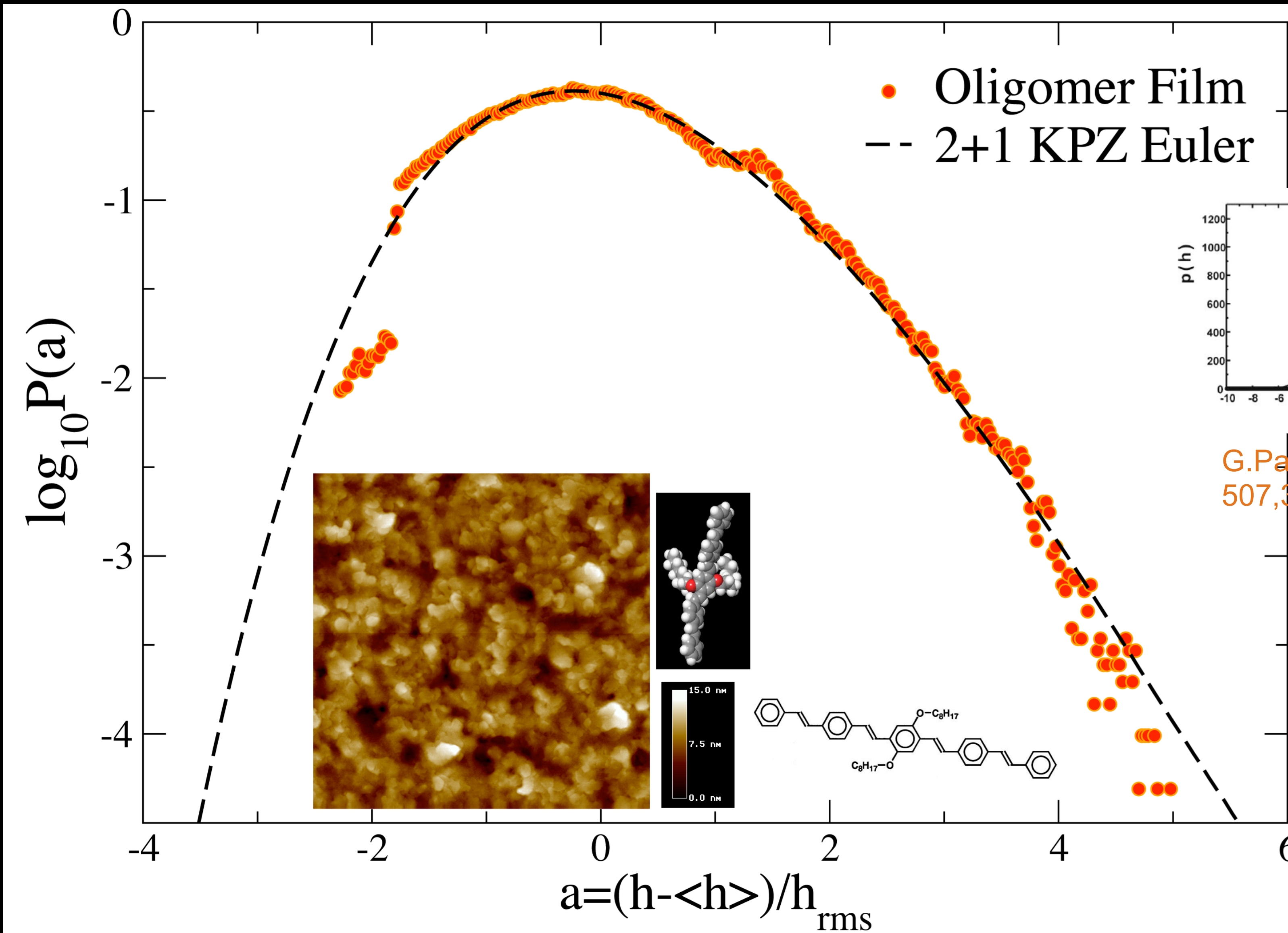
(unit variance, zero mean)

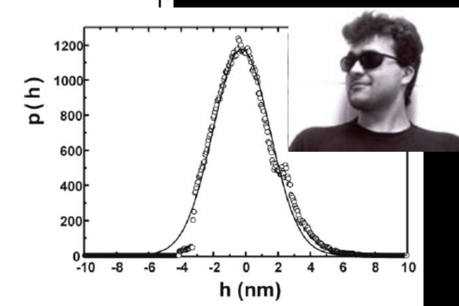


THH- PRL**109**,170602(2012);
Brazil- PRE**87**, 040102 (2013).

2+1 KPZ CLASS HD:
Thin Film Expt [NL]-

*Almeida-PRB89,045309(2014)
THH/GP-EPL105,50001(2014)





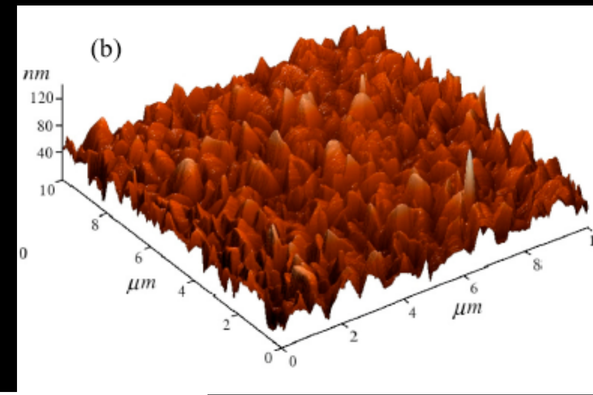
G.Palasantzas,Surf.Sci. 507,357(2002)

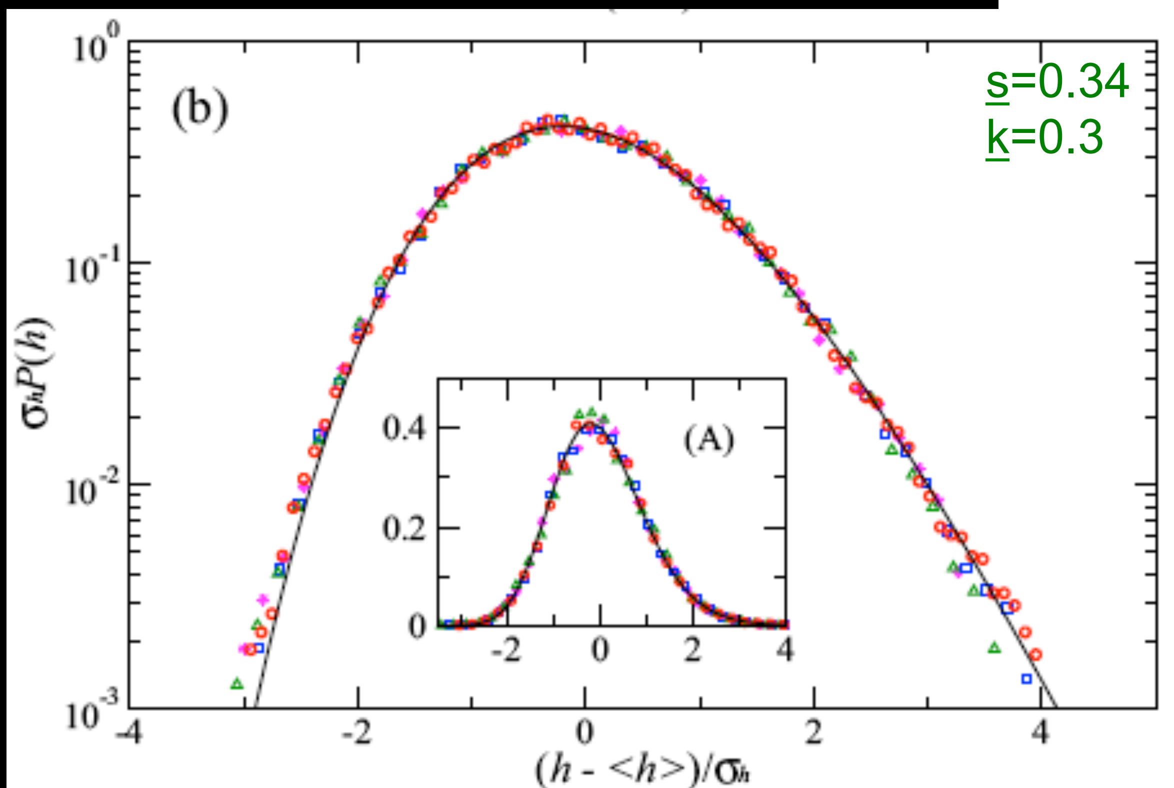
2+1 KPZ CLASS HD:

Thin Film Expt [BRASIL]

Almeida-PRB89,045309(2014)

CdTe/Si Semiconductor Film:

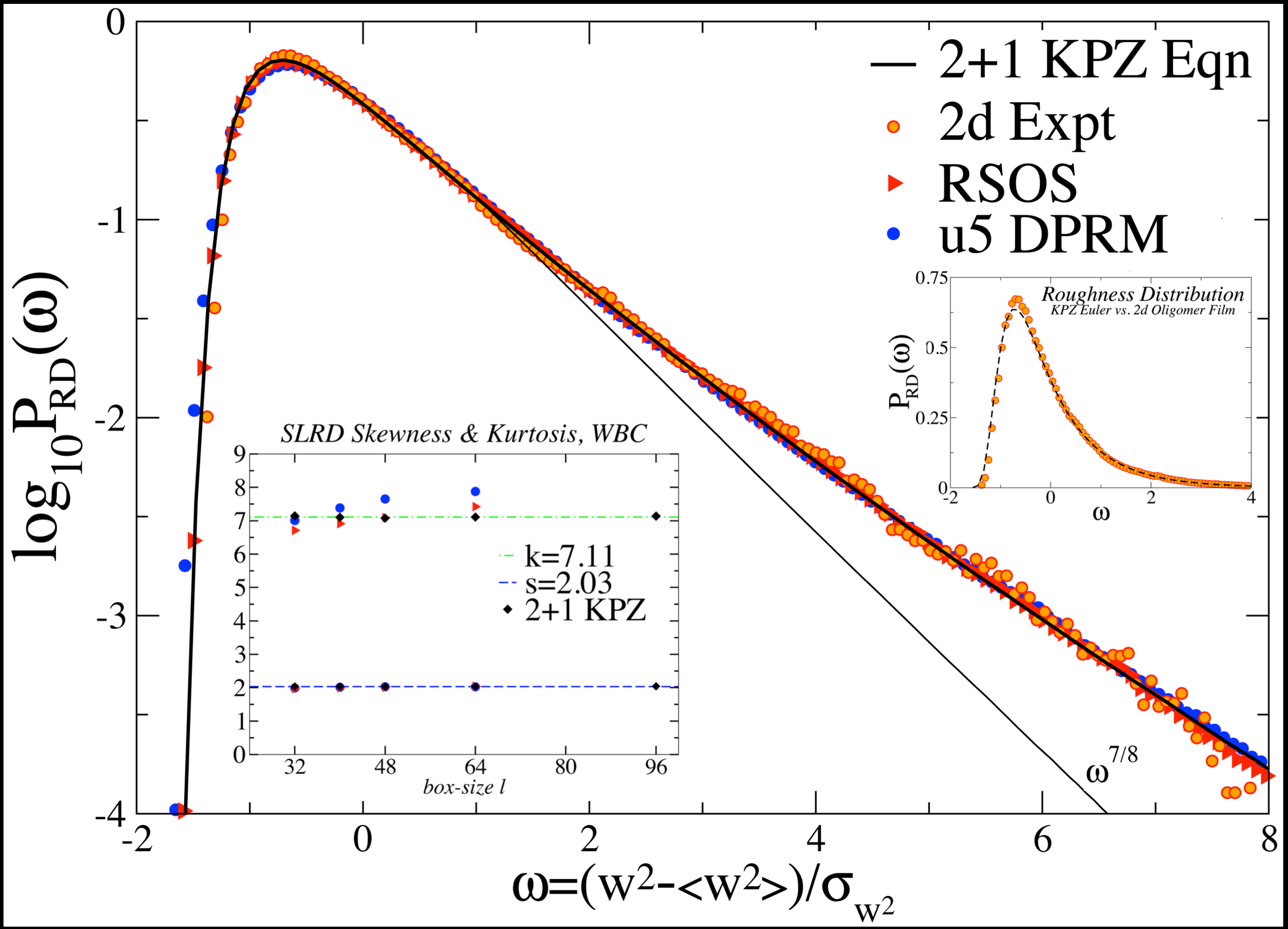




Squared Local Roughness Distribution:

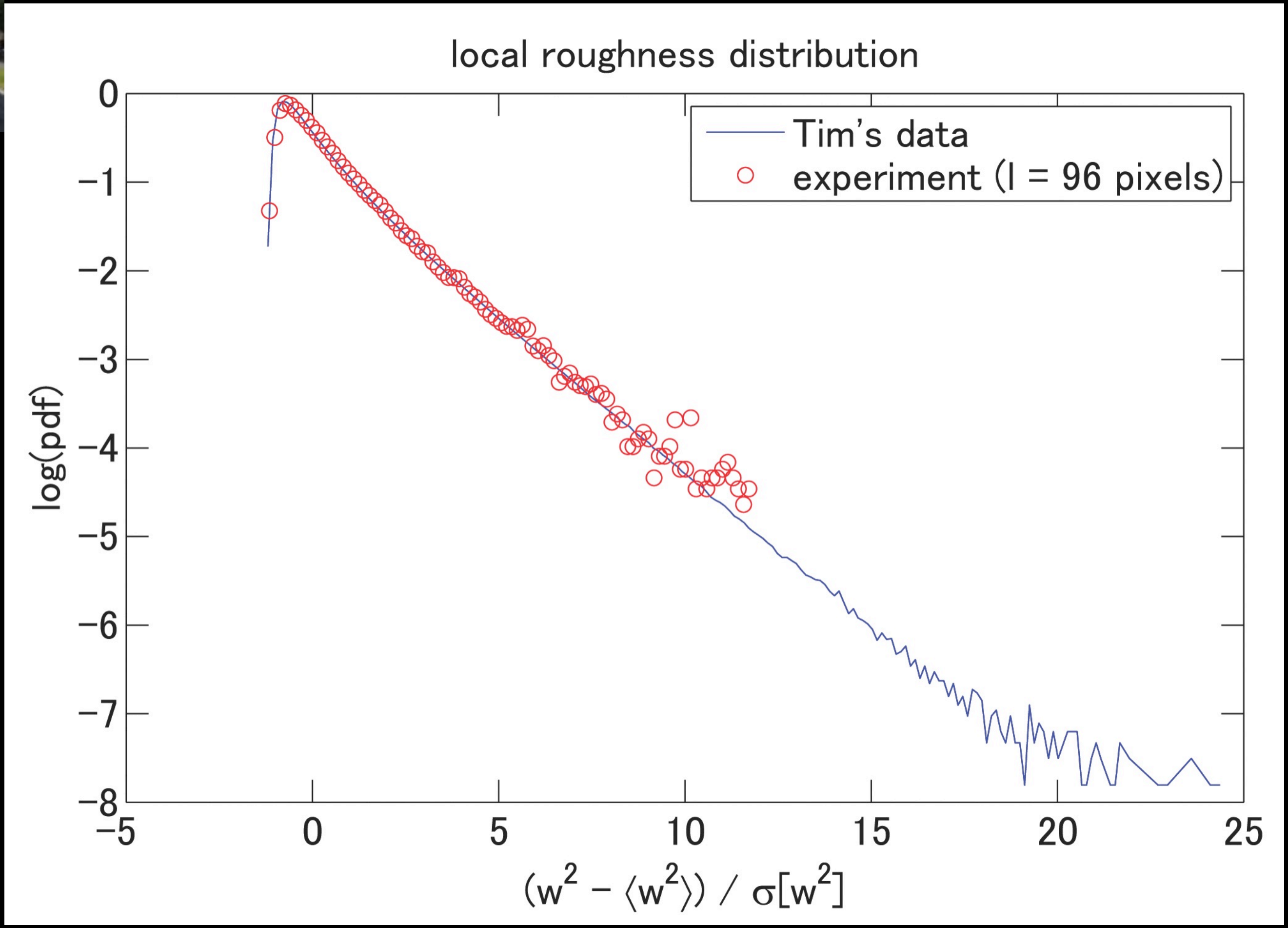
(WBC, not PBC!)

THH & Palasantzas, EPL105,50001(2014)
Almeida,(2014); Z. Racz, PRE50,3530(1994)



1+1 KPZ Class: SLRD

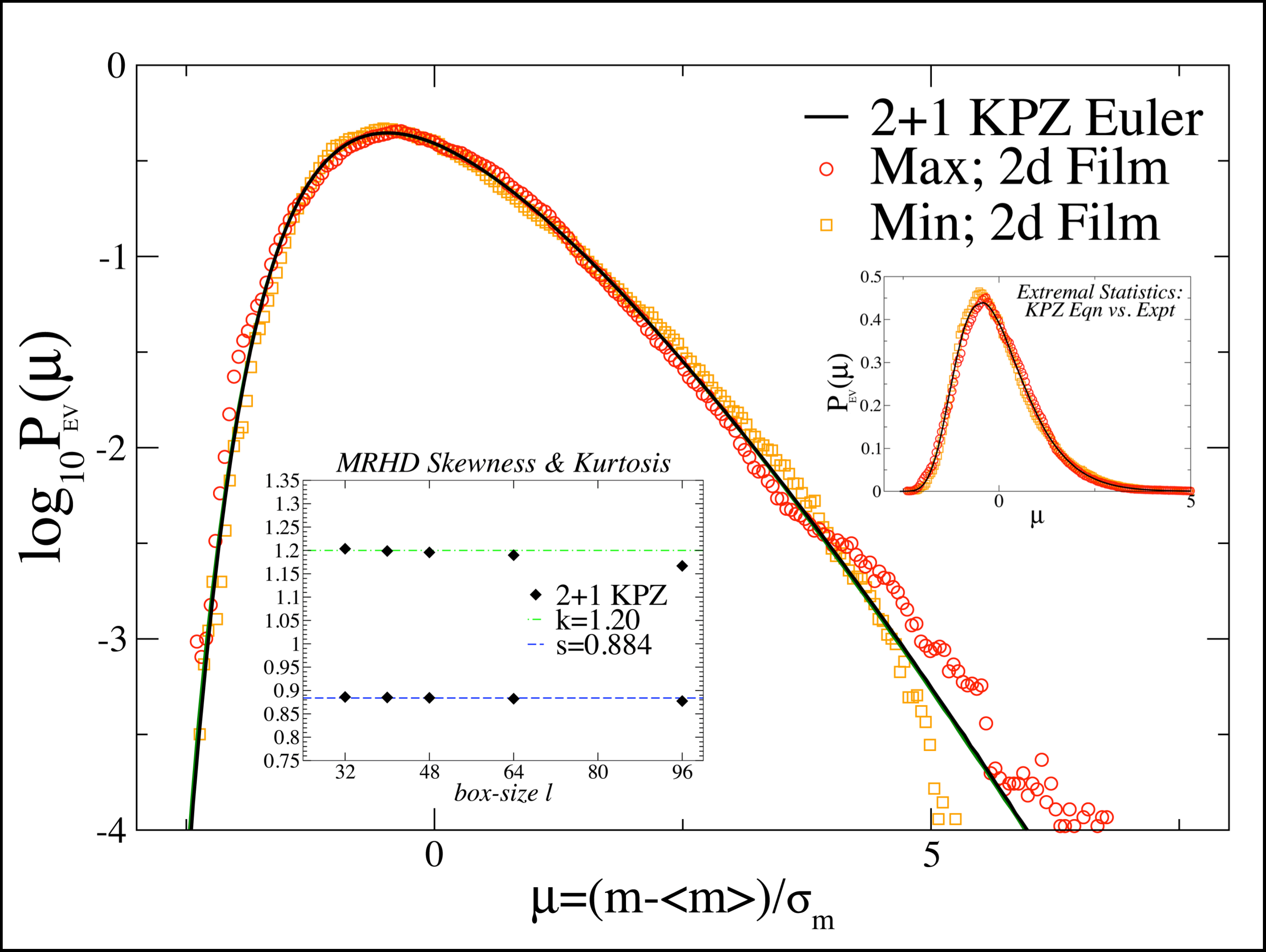
(Takeuchi & Sano- Liquid Crystal Expt vs. KPZ Euler...)



Reality check....

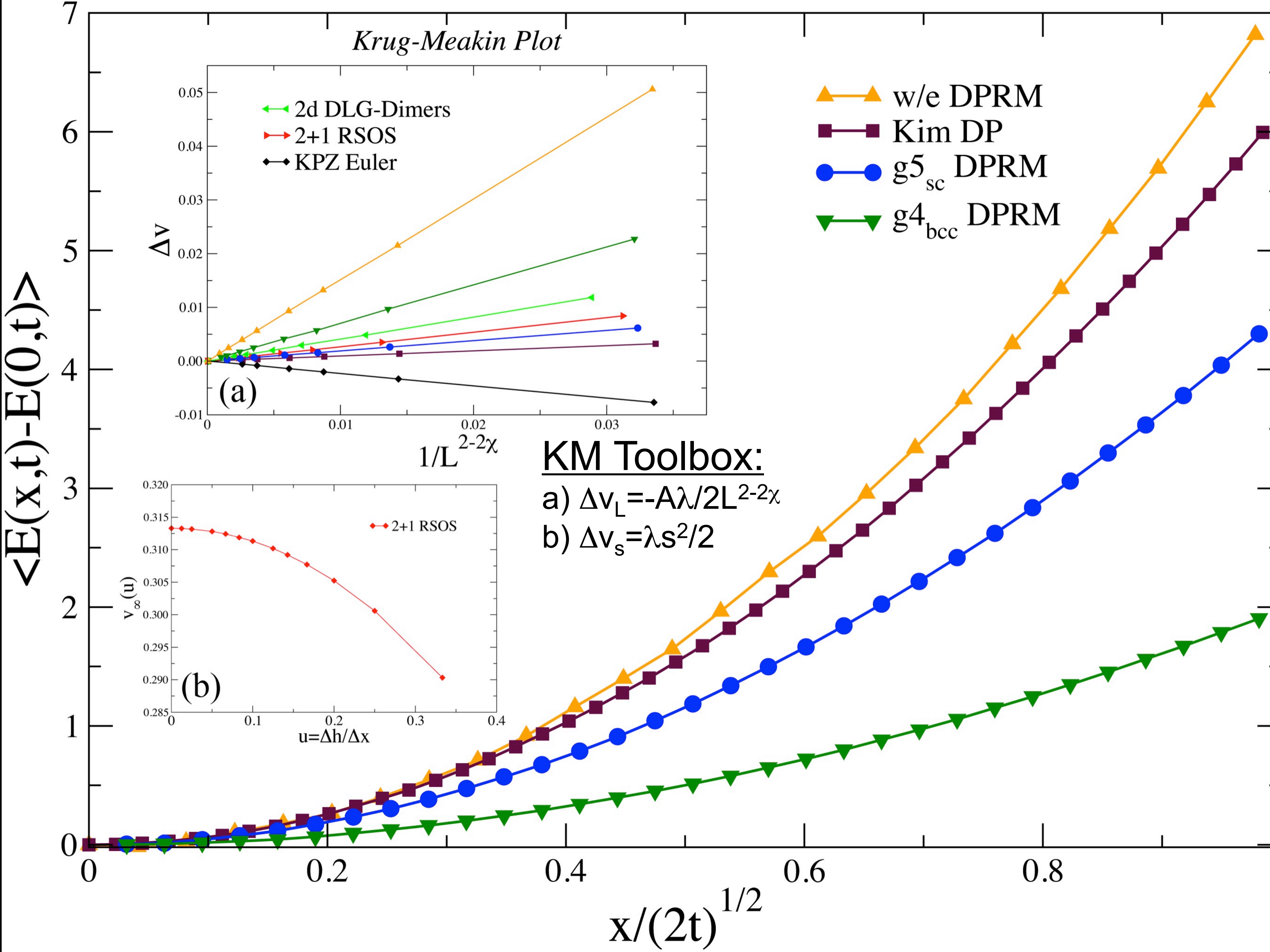
Extremal Height Distributions:
(WBC)

THH & Palasantzas, EPL105,50001(2014).



2+1 KPZ

Universal Limit Distribution*



Krug-Meakin Plot
2d DLG-Dimers
2+1 RSOS
KPZ Euler
Δv
1/L^{2-2χ}
(a)
2+1 RSOS
v_∞(u)
u=Δh/Δx
(b)
w/e DPRM
Kim DP
g5_sc DPRM
g4_bcc DPRM
KM Toolbox:
a) Δv_L=-Aλ/2L^{2-2χ}
b) Δv_s=λs²/2
<E(x,t)-E(0,t)>
x/(2t)^{1/2}

TABLE I. 2 + 1 KPZ model parameters, *point-plane* DPRM geometry; equivalently, KPZ stochastic growth from a flat substrate.

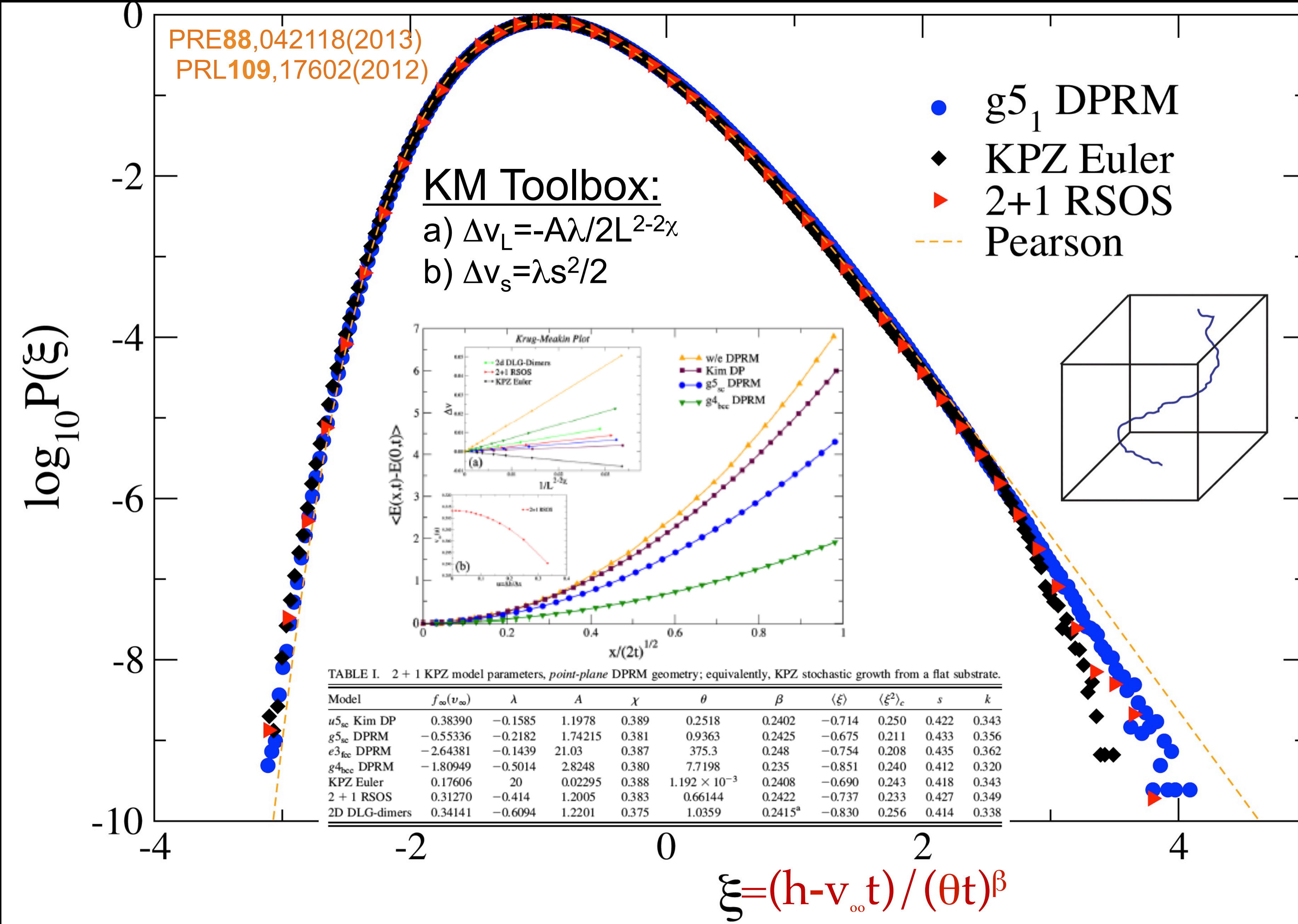
Model	$f_\infty(v_\infty)$	λ	A	χ	θ	β	$\langle\xi\rangle$	$\langle\xi^2\rangle_c$	s	k
$u5_{sc}$ Kim DP	0.38390	−0.1585	1.1978	0.389	0.2518	0.2402	−0.714	0.250	0.422	0.343
$g5_{sc}$ DPRM	−0.55336	−0.2182	1.74215	0.381	0.9363	0.2425	−0.675	0.211	0.433	0.356
$e3_{fcc}$ DPRM	−2.64381	−0.1439	21.03	0.387	375.3	0.248	−0.754	0.208	0.435	0.362
$g4_{bcc}$ DPRM	−1.80949	−0.5014	2.8248	0.380	7.7198	0.235	−0.851	0.240	0.412	0.320
KPZ Euler	0.17606	20	0.02295	0.388	1.192×10^{-3}	0.2408	−0.690	0.243	0.418	0.343
2 + 1 RSOS	0.31270	−0.414	1.2005	0.383	0.66144	0.2422	−0.737	0.233	0.427	0.349
2D DLG-dimers	0.34141	−0.6094	1.2201	0.375	1.0359	0.2415[a]	−0.830	0.256	0.414	0.338

[a]Ref. [15]

Devil in the details...

λ, A, $\theta = A^{1/\chi}\lambda$

2+1 KPZ CLASS: Limit Distribution*



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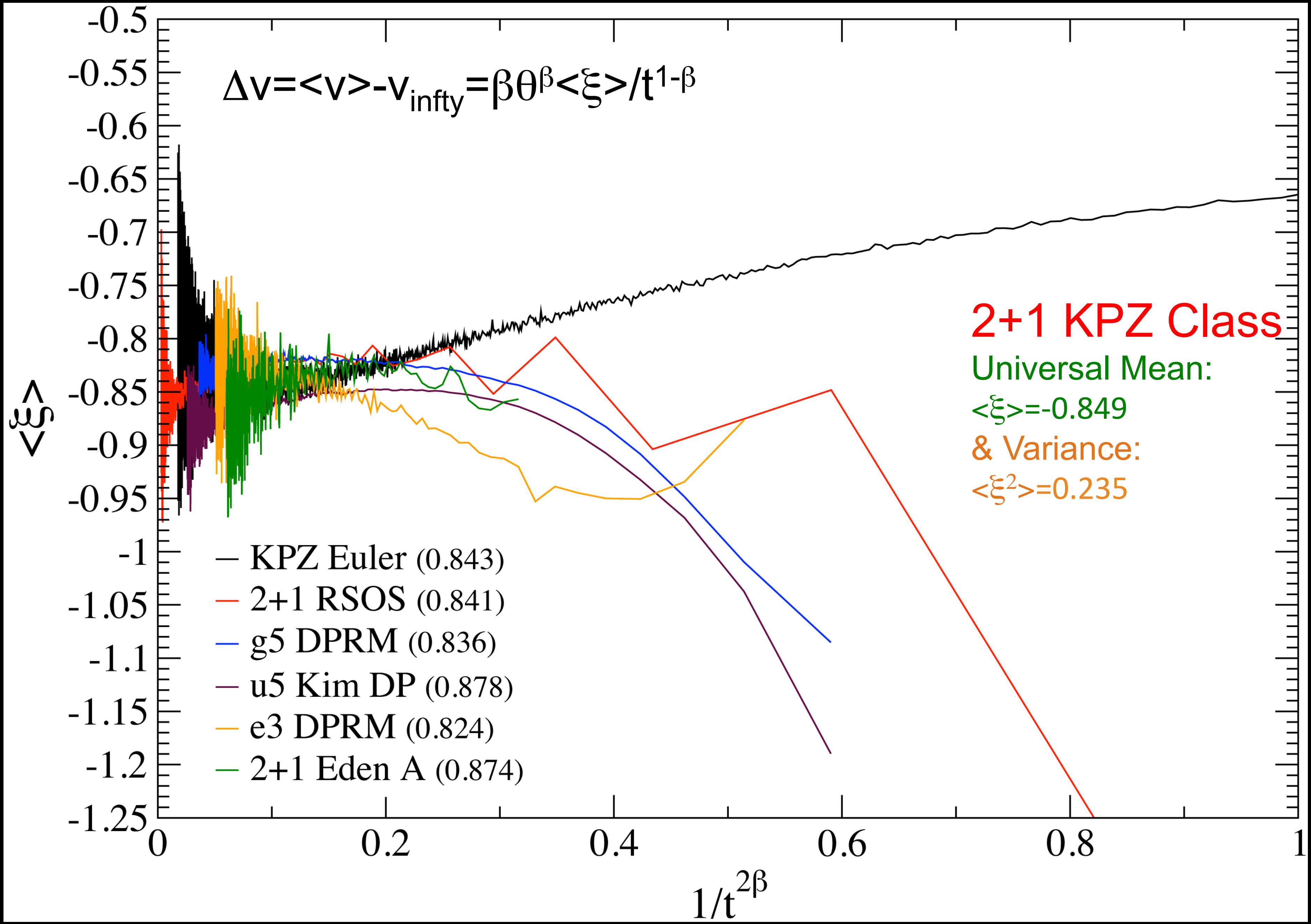
Model	$f_\infty(v_\infty)$	λ	A	χ	θ	β	$\langle\xi\rangle$	$\langle\xi^2\rangle_c$	s	k
$u5_{sc}$ Kim DP	0.38390	−0.1585	1.1978	0.389	0.2518	0.2402	−0.714	0.250	0.422	0.343
$g5_{sc}$ DPRM	−0.55336	−0.2182	1.74215	0.381	0.9363	0.2425	−0.675	0.211	0.433	0.356
$e3_{fcc}$ DPRM	−2.64381	−0.1439	21.03	0.387	375.3	0.248	−0.754	0.208	0.435	0.362
$g4_{bcc}$ DPRM	−1.80949	−0.5014	2.8248	0.380	7.7198	0.235	−0.851	0.240	0.412	0.320
KPZ Euler	0.17606	20	0.02295	0.388	1.192×10^{-3}	0.2408	−0.690	0.243	0.418	0.343
2 + 1 RSOS	0.31270	−0.414	1.2005	0.383	0.66144	0.2422	−0.737	0.233	0.427	0.349
2D DLG-dimers	0.34141	−0.6094	1.2201	0.375	1.0359	0.2415[a]	−0.830	0.256	0.414	0.338

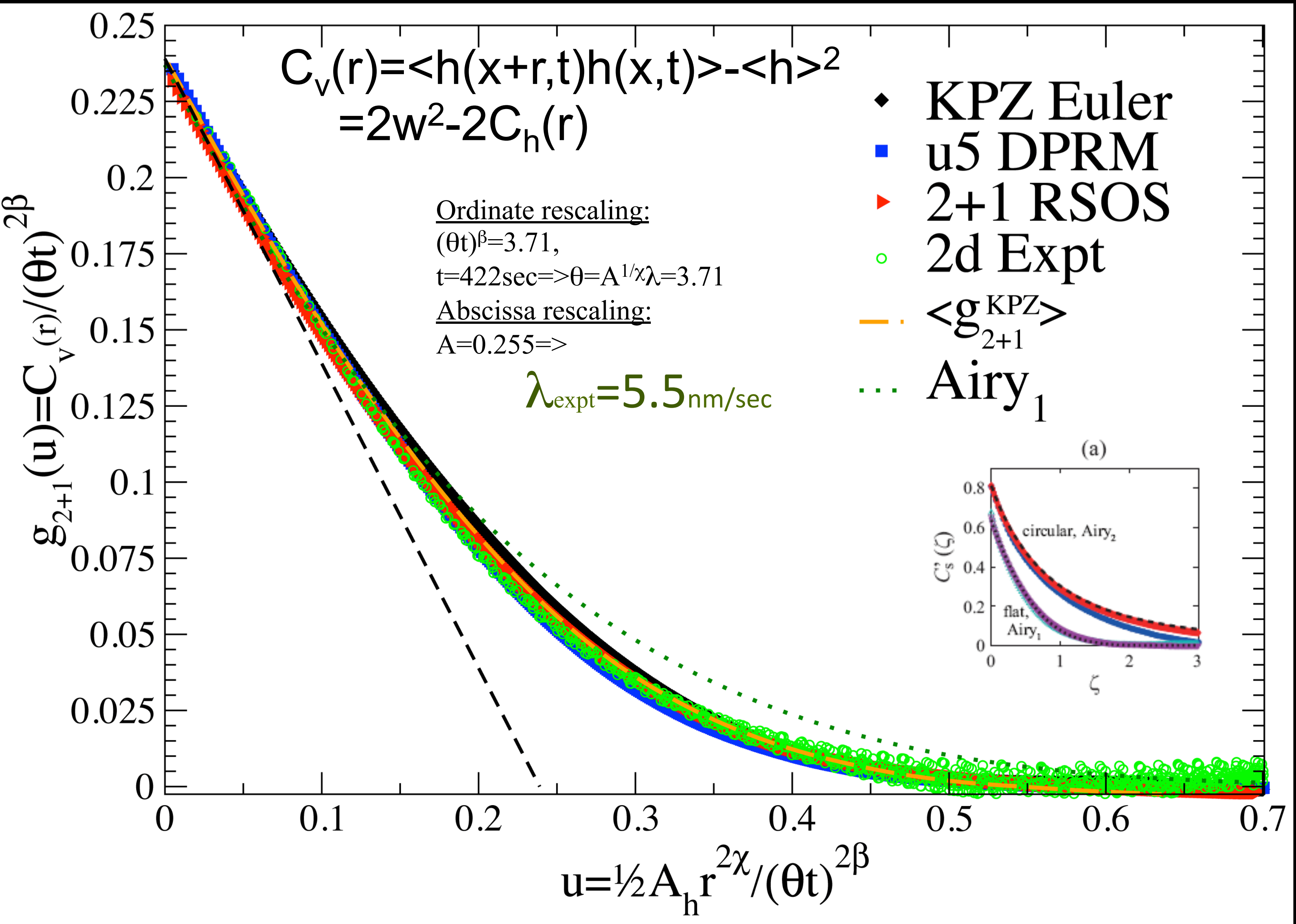


Δv=<v>-v_infty=βθ^β<ξ>/t^(1-β)
2+1 KPZ Class
Universal Mean:
<ξ>=-0.849
& Variance:
<ξ²>=0.235
KPZ Euler (0.843)
2+1 RSOS (0.841)
g5 DPRM (0.836)
u5 Kim DP (0.878)
e3 DPRM (0.824)
2+1 Eden A (0.874)
<ξ>
1/t^2β
-0.5
-0.55
-0.6
-0.65
-0.7
-0.75
-0.8
-0.85
-0.9
-0.95
-1
-1.05
-1.1
-1.15
-1.2
-1.25
0
0.2
0.4
0.6
0.8
1

2+1 KPZ

Universal Correlators*

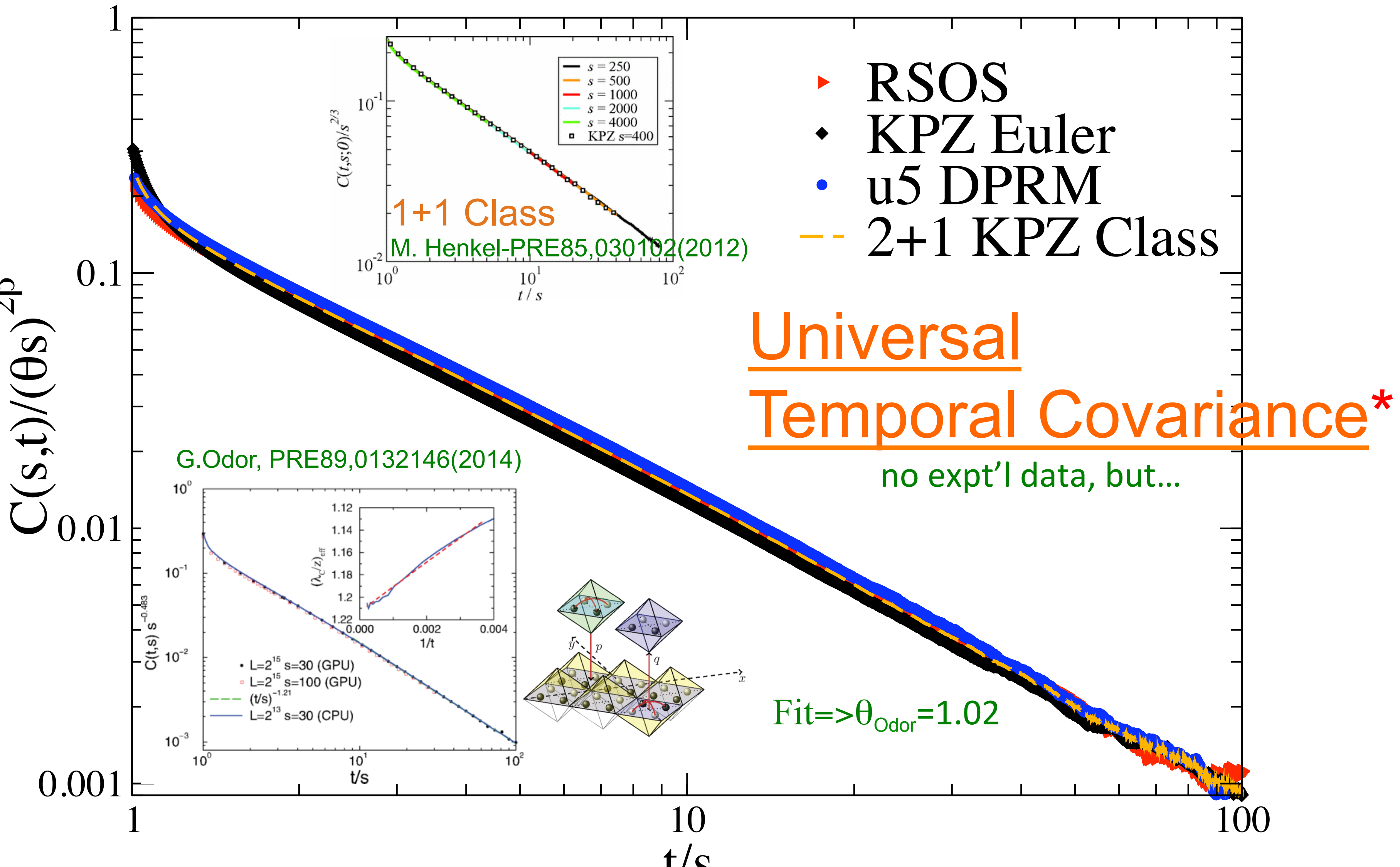
2+1 KPZ Spatial Covariance*



Two-Time Autocorrelator: C(t,s)=<h(t)h(s)>-<h(t)><h(s)>

$=s^{2\beta}F_c(t/s)$

z2age.f: its*dt=250*0.02=>s=5; r2age.f: s=100; L=10k; nr=28



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neous growth velocity; analogously, $f_\infty = \langle dF/dt \rangle$, the DPRM free energy per unit length. It is the distribution $P(\xi)$ which lies at the heart of 2 + 1 KPZ class universality, and the matter demands, in addition to knowledge of θ, a precise determination of KPZ-DPRM v_∞/f_∞. To this end, we have relied heavily upon a Krug-Meakin [20] finite-size scaling analysis which, by virtue of a truncated Fourier sum over modes, reveals that the KPZ growth velocity in a system of finite-size L suffers a small shift from its true asymptotic value: $\Delta v \equiv \langle dh/dt \rangle - v_\infty = -\frac{1}{2} A\lambda / L^{2-2\chi}$; for the DPRM problem, the corresponding

Spohn conjecture above. Ultimately, it follows from the fact that at early times with conical IC, the KPZ nonlinearity dominates, generating Cole-Hopf paraboloids with small superposed distortions arising from the additive KPZ noise term. While such noise is visible for each individual run, ensemble averaging produces a smooth parabolic profile—see Fig 2, proper, which follows from 10^4 realizations of our DPRM random energy landscape. Alternatively, for the KPZ stochastic growth models, such as 2 + 1 RSOS, we study the tilt-dependent growth velocity [23], Fig. 2(b). For 2D driven dimers, A is known

TABLE I. 2 + 1 KPZ model parameters, *point-plane* DPRM geometry; equivalently, KPZ stochastic growth from a flat substrate.

Model	$f_\infty(v_\infty)$	λ	A	χ	θ	β	$\langle\xi\rangle$	$\langle\xi^2\rangle_c$	s	k
$u5_{\text{sc}}$ Kim DP	0.38390	−0.1585	1.1978	0.389	0.2518	0.2402	−0.714	0.250	0.422	0.343
$g5_{\text{sc}}$ DPRM	−0.55336	−0.2182	1.74215	0.381	0.9363	0.2425	−0.675	0.211	0.433	0.356
$e3_{\text{fcc}}$ DPRM	−2.64381	−0.1439	21.03	0.387	375.3	0.248	−0.754	0.208	0.435	0.362
$g4_{\text{bcc}}$ DPRM	−1.80949	−0.5014	2.8248	0.380	7.7198	0.235	−0.851	0.240	0.412	0.320
KPZ Euler	0.17606	20	0.02295	0.388	1.192×10^{-3}	0.2408	−0.690	0.243	0.418	0.343
2 + 1 RSOS	0.31270	−0.414	1.2005	0.383	0.66144	0.2422	−0.737	0.233	0.427	0.349
2D DLG-dimers	0.34141	−0.6094	1.2201	0.375	=>1.0359*	0.2415[a]	−0.830	0.256	0.414	0.338

[a]Ref. [15] Kelling&Odor-PRE84,061150(2011)

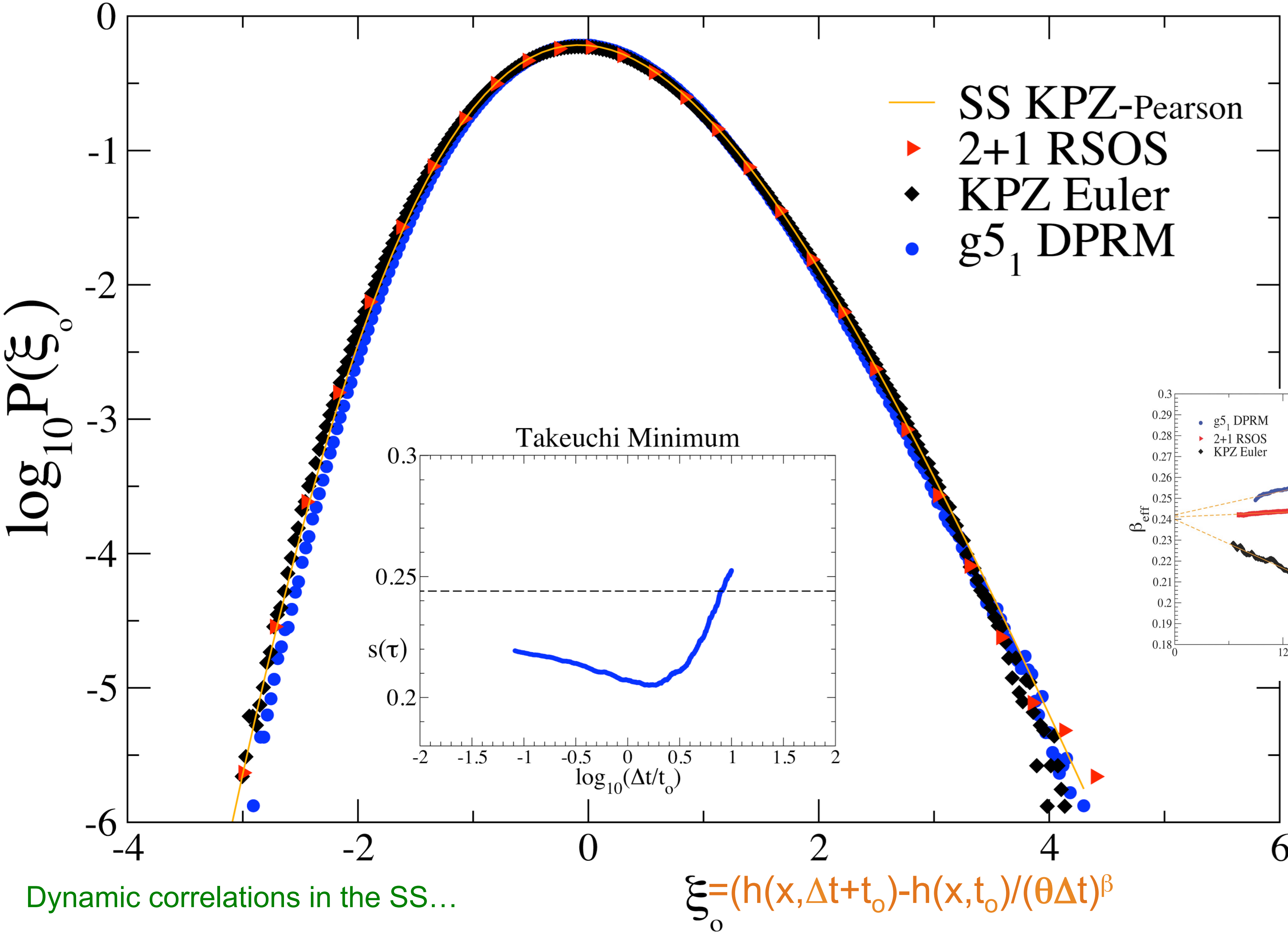
Devil in the details...

λ, A, θ=$A^{1/\chi}\lambda$

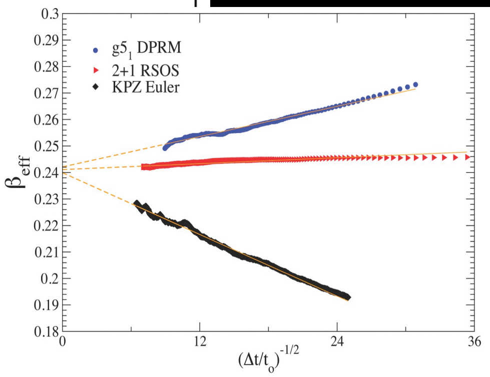
2+1 Stationary-State KPZ*

(higher-dimensional analog Baik-Rains)

Universal Variance-
$\langle \xi_o^2 \rangle = 0.464$



Nonperturbative Functional RG-
$\langle \xi_o^2 \rangle = 0.462$
Kloss, Canet, Wschebor
PRE86,051124 (2012)



Dynamic correlations in the SS…

3d Radial/pt-pt KPZ Limit Distribution:

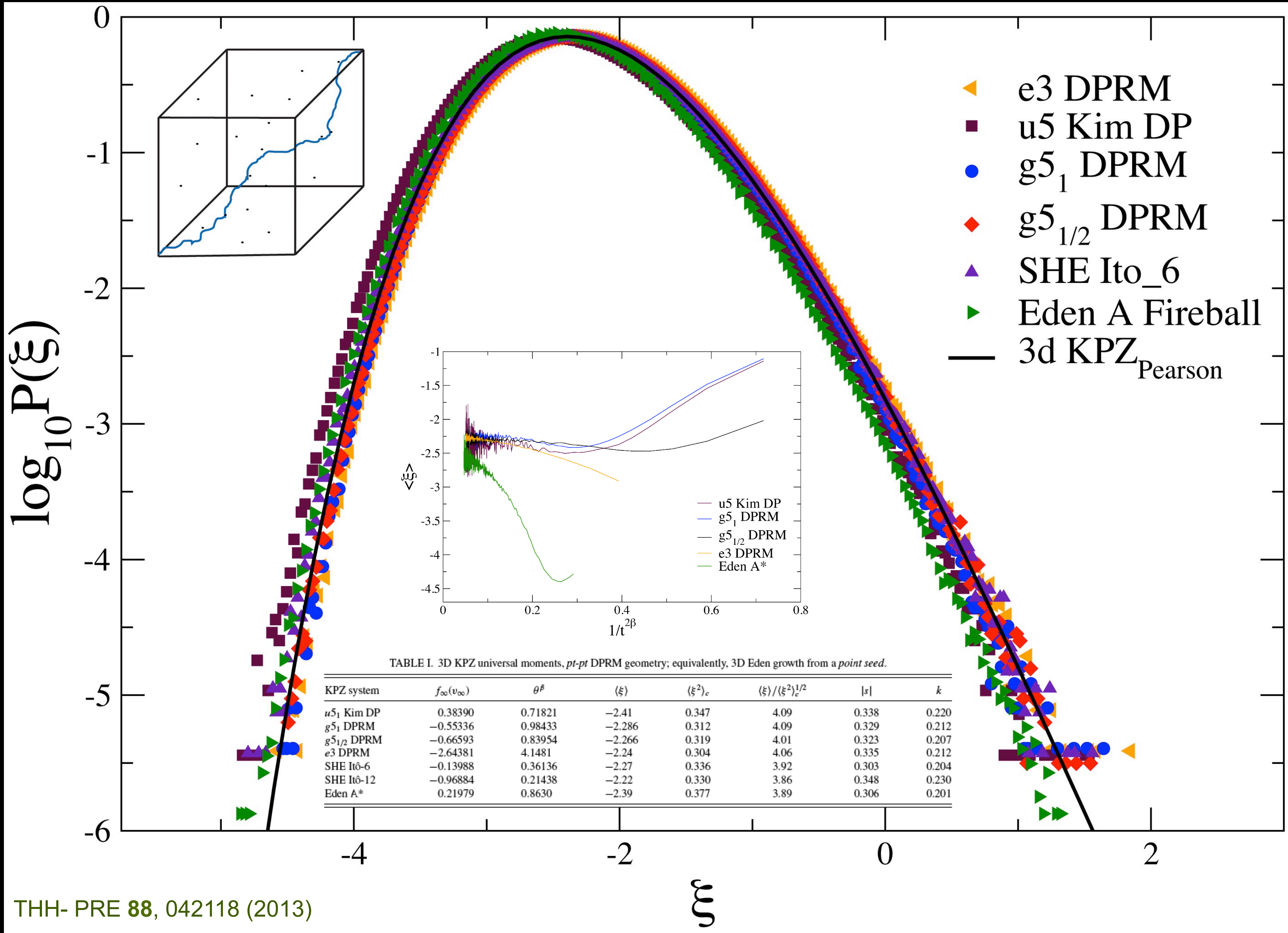


TABLE I. 3D KPZ universal moments, *pt-pt* DPRM geometry; equivalently, 3D Eden growth from a *point seed*.

KPZ system	$f_\infty(v_\infty)$	θ^β	$\langle\xi\rangle$	$\langle\xi^2\rangle_c$	$\langle\xi\rangle/\langle\xi^2\rangle_c^{1/2}$	$\lvert s\rvert$	k
$u5_1$ Kim DP	0.38390	0.71821	−2.41	0.347	4.09	0.338	0.220
$g5_1$ DPRM	−0.55336	0.98433	−2.286	0.312	4.09	0.329	0.212
$g5_{1/2}$ DPRM	−0.66593	0.83954	−2.266	0.319	4.01	0.323	0.207
$e3$ DPRM	−2.64381	4.1481	−2.24	0.304	4.06	0.335	0.212
SHE Itô-6	−0.13988	0.36136	−2.27	0.336	3.92	0.303	0.204
SHE Itô-12	−0.96884	0.21438	−2.22	0.330	3.86	0.348	0.230
Eden A*	0.21979	0.8630	−2.39	0.377	3.89	0.306	0.201

THH- PRE **88**, 042118 (2013)

2+1 KPZ Class:

3+ Universal PDFs, 2 Correlators, & KM Toolbox
=>Rich, Ripe, & Ready to go…

Arigato!

2+1 KPZ NUMERICS: THH- PRL**109**,170602 (2012)
PRE**88**,042118 (2013)
PRE**89**,010103R (2014) w/LunaLin

2+1 KPZ Expt: Almeida-PRB**89**,045309 (2014)
Palasantzas-EPL**105**,50001 (2014)

KT